Computing All Approximate Enhanced Covers with the Hamming Distance

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1. Introduction
    - About the Topics
2. Preliminaries
    - The Notions
    - Automata
3. The Algorithm
    - Basic Idea
    - Time and Space Complexity
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    - Future Work

Background

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- quite restrictive
- attempts to introduce more relaxed regularities
- enhanced covers, approximate enhanced covers

Example

$\boldsymbol{x} = \texttt{aabaaccaabaa}$

Definition

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1. a border of $\boldsymbol{x}$

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$\boldsymbol{x} = \texttt{aabaaccaabaa}$ $\quad k = 1$

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	a	aa	aabaa
covered	N/A	12	10

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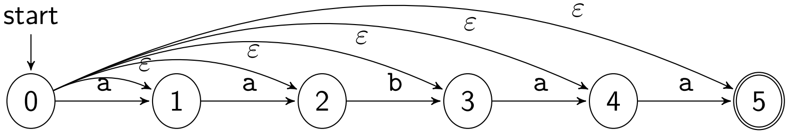
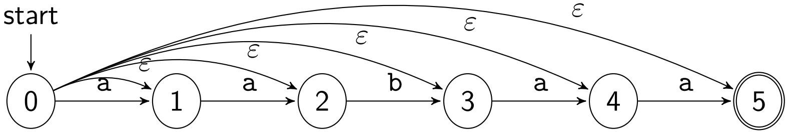
Suffix automaton

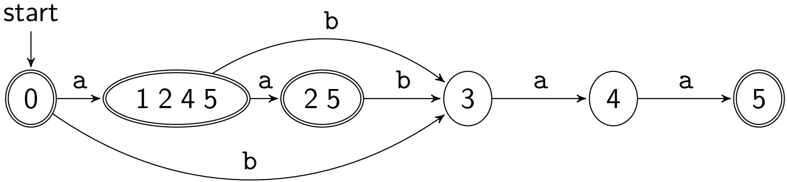
Nondeterministic



Suffix automaton

Deterministic





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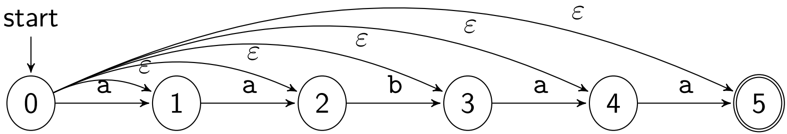
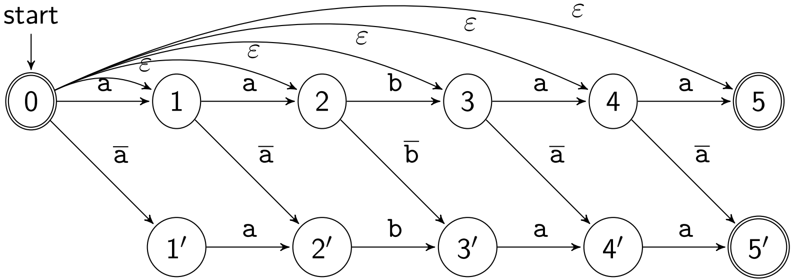


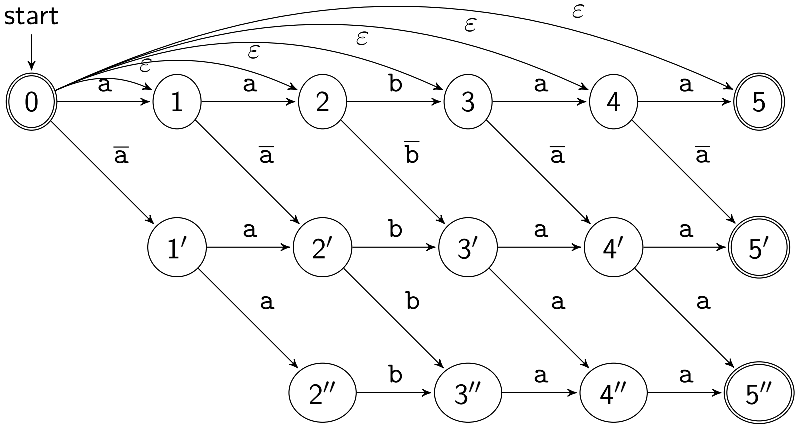
Suffix automaton

Nondeterministic Hamming $k = 1$



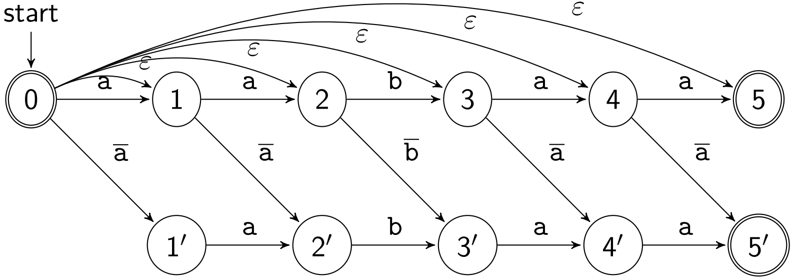
Suffix automaton

Nondeterministic Hamming $k = 2$



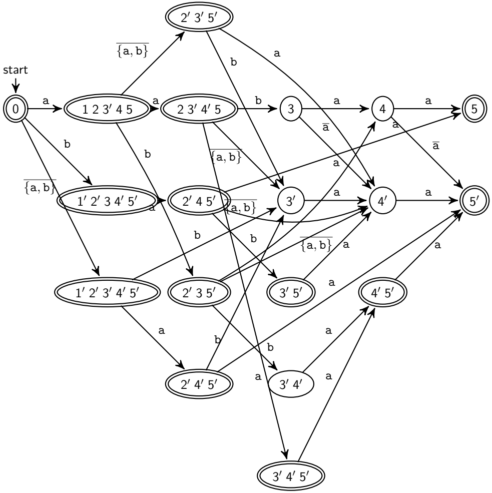
Suffix automaton

Nondeterministic Hamming $k = 1$



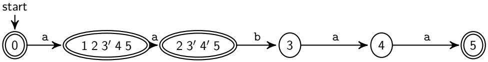
Suffix automaton

Deterministic, Hamming $k = 1$



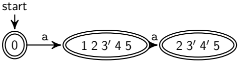
Suffix automaton

Backbone, Hamming $k = 1$



Suffix automaton

Backbone multiple front end, Hamming $k = 1$



Outline of the algorithm

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2. Construct backbone using subset construction
3. For each appropriate state, count covered positions and remember

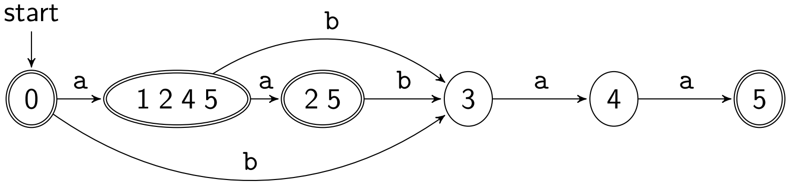
- Space complexity: $\mathcal{O}(n)$
- Time complexity: $\mathcal{O}(n^2)$

Construction of the backbone

Backbone only

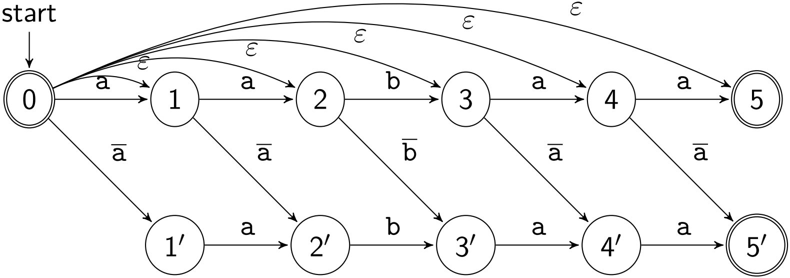
Just backbone is constructed, nothing else is needed

Example



Construction of the backbone

Nondeterministic automaton is not needed



Construction of the backbone

Nondeterministic automaton is not needed

Construction of the first backbone state:

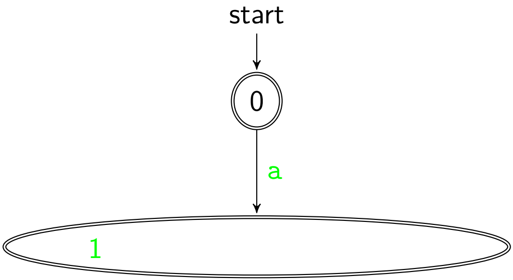
```
$q_1$ is a state;
for $i \in 1..|\mathbf{x}|$ do
    $e$ is a d-subset element such that depth($e$) $\leftarrow i$;
    if $\mathbf{x}[1] = \mathbf{x}[i]$ then
        level($e$) $\leftarrow 0$;
        append $e$ to $q_1$
    else if $k > 0$ then
        level($e$) $\leftarrow 1$;
        append $e$ to $q_1$
    end
end
```

Construction of the backbone

Nondeterministic automaton is not needed

Example

	1	2	3	4	5	6	7	8	9	10	11	12
$\boldsymbol{x} =$	a	a	b	a	a	c	c	a	a	b	a	a



Construction of the backbone

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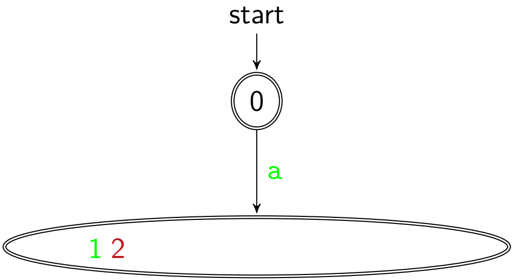
Example

$$\begin{array}{ccccccccccccc} & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 \\ \boldsymbol{x} = & \mathsf{a} & \mathsf{a} & \mathsf{b} & \mathsf{a} & \mathsf{a} & \mathsf{c} & \mathsf{c} & \mathsf{a} & \mathsf{a} & \mathsf{b} & \mathsf{a} & \mathsf{a} \end{array}$$



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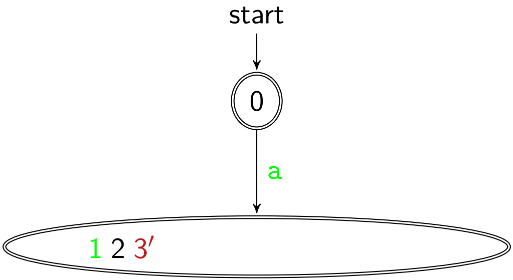

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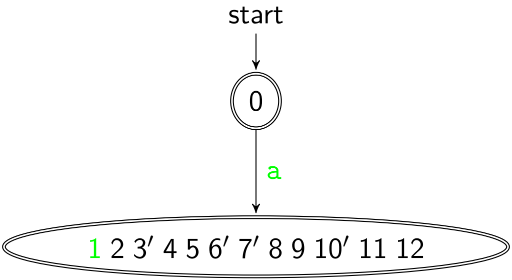

Construction of the backbone

Nondeterministic automaton is not needed

Construction of a next backbone state:

```
for $i \in 2..|\boldsymbol{x}|$ do
    for $e_p \in q_p$ do
        if $\mathrm{depth}(e_p) < |\boldsymbol{x}|$ then
            $e_n$ is new d-subset element;
            $\mathrm{depth}(e_n) \leftarrow \mathrm{depth}(e_p) + 1$;
            if $\boldsymbol{x}[i] = \boldsymbol{x}[\mathrm{depth}(e_n)]$ then
                $\mathrm{level}(e_n) \leftarrow \mathrm{level}(e_p)$;
                append $e_n$ to $q_n$;
            else if $\mathrm{level}(e_p) < k$ then
                $\mathrm{level}(e_n) \leftarrow \mathrm{level}(e_p) + 1$;
                append $e_n$ to $q_n$;
            end
        end
    end
end
```

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Example

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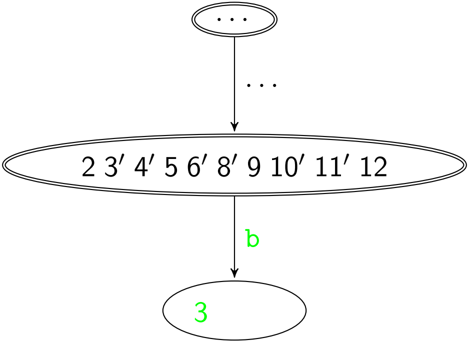
$\boldsymbol{x} =$	a	a	b	a	a	c	c	a	a	b	a	a



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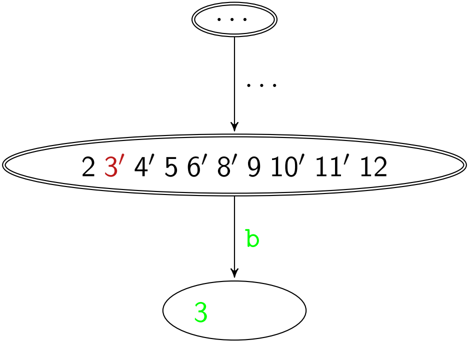

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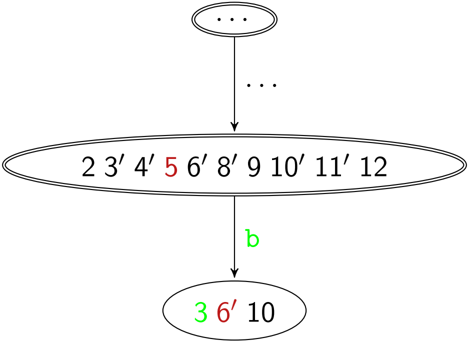

Construction of the backbone

Removal of states

Example

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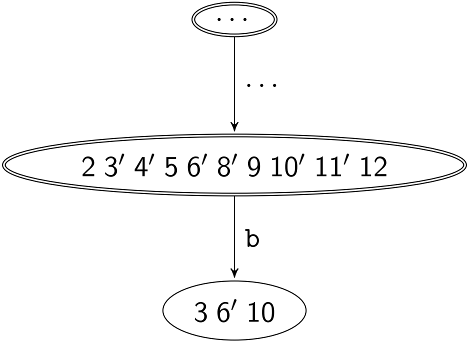
$\boldsymbol{x} =$	a	a	b	a	a	c	c	a	a	b	a	a



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    - based on finite automata
    - $\mathcal{O}(n)$ space and $\mathcal{O}(n^2)$ time

- reduce time complexity

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- variants of the problem

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- *k*-approximate enhanced cover array