Generating All Minimal Petri Net Unsolvable Binary Words

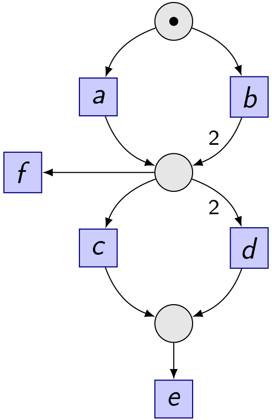
Kamila Barylska
Marcin Piątkowski
Łukasz Mikulski

Evgeny Erofeev

Nicolaus Copernicus University
Toruń, Poland

Carl von Ossietzky Universität
Oldenburg, Germany

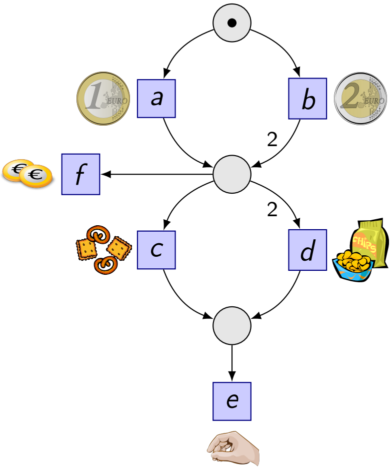
20th Prague Stringology Conference 2016



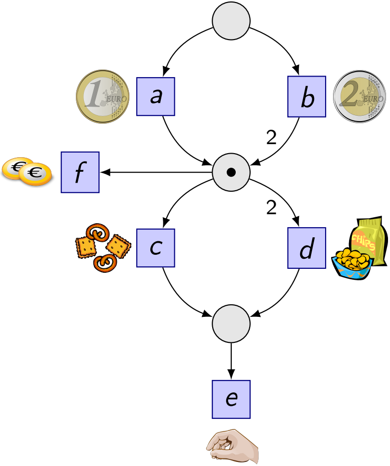
Petri Net

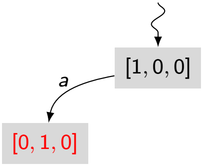
A tuple $N = (P, T, F, M_0)$, where:

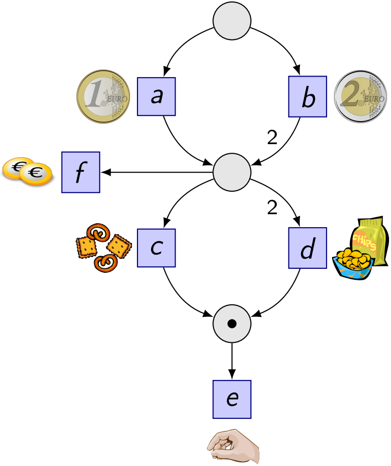
- P – finite set of places
- T – finite set of transitions
- F – flow function
  $F\colon \Big((P \times T) \cup (T \times P)\Big) \to I\!N$
- M_0 – initial marking

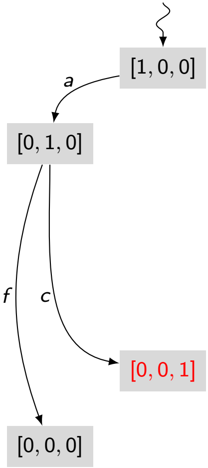


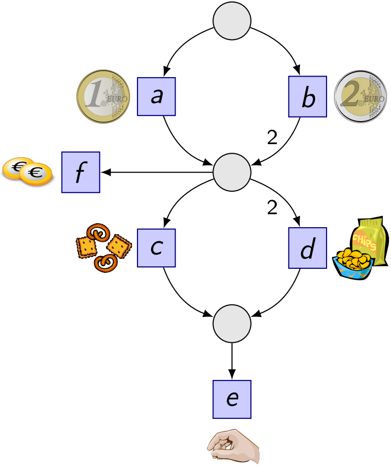
[1, 0, 0]

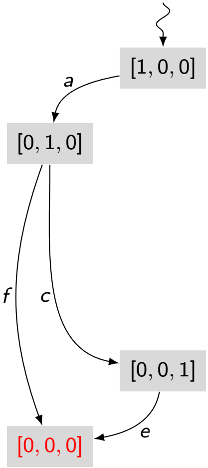

a
b
2
f
2
c
d
e

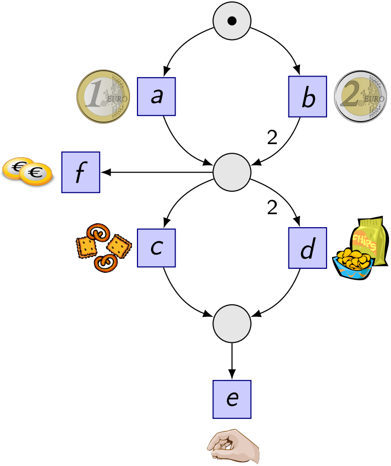

[1, 0, 0]
a
[0, 1, 0]

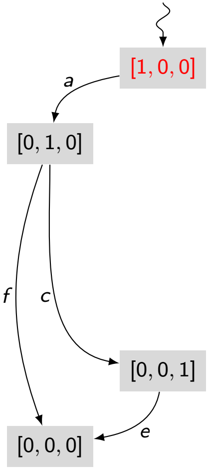

a
b
2
f
2
c
d
e

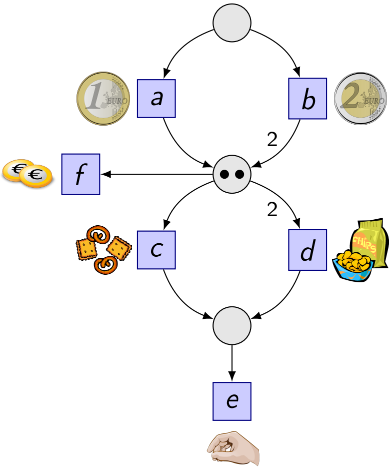

[1, 0, 0]
a
[0, 1, 0]
f
c
[0, 0, 1]
[0, 0, 0]

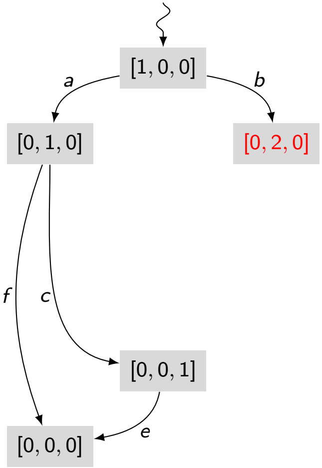

a
b
2
f
2
c
d
e

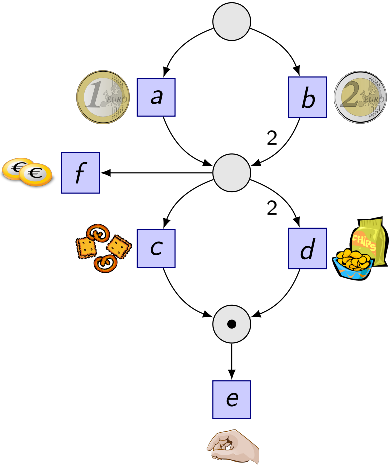

[1, 0, 0]
a
[0, 1, 0]
f
c
[0, 0, 1]
e
[0, 0, 0]

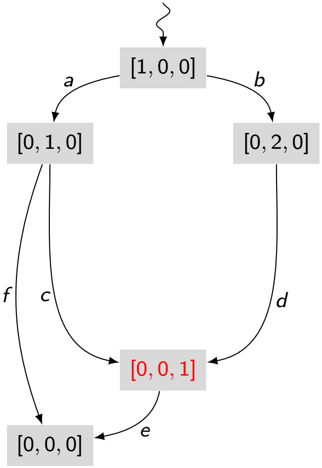

a
b
2
f
2
c
d
e

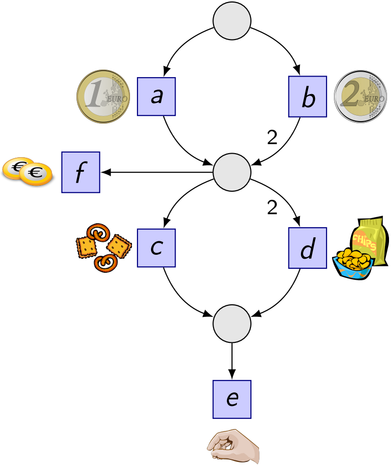

[1, 0, 0]
a
[0, 1, 0]
f
c
[0, 0, 1]
e
[0, 0, 0]


a
b
2
f
c
d
2
e

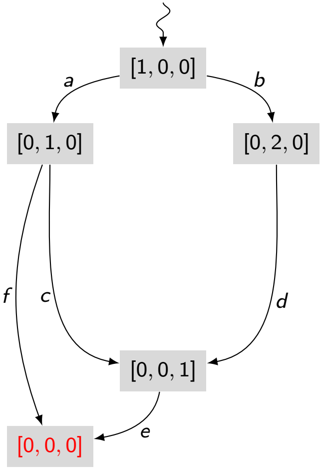

[1, 0, 0]
a
b
[0, 1, 0]
[0, 2, 0]
f
c
[0, 0, 1]
e
[0, 0, 0]


a
b
2
f
2
c
d
e

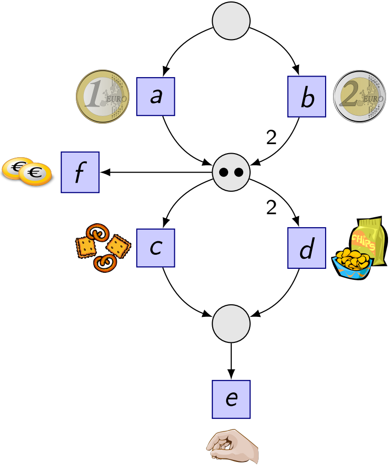

[1, 0, 0]
a
b
[0, 1, 0]
[0, 2, 0]
f
c
d
[0, 0, 1]
e
[0, 0, 0]



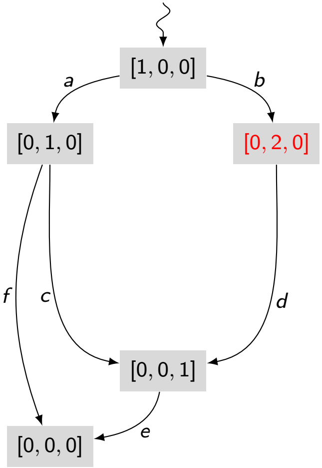
a
b
2
f
2
c
d
e



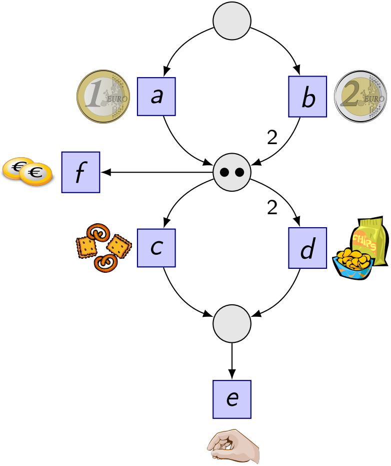
[1, 0, 0]
a
b
[0, 1, 0]
[0, 2, 0]
f
c
d
[0, 0, 1]
e
[0, 0, 0]

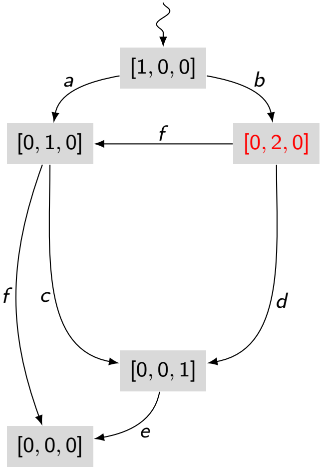


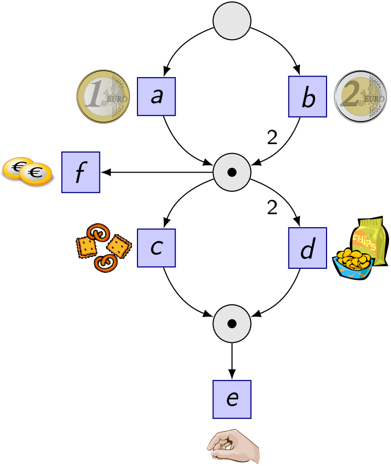
a
b
2
f
2
c
d
e

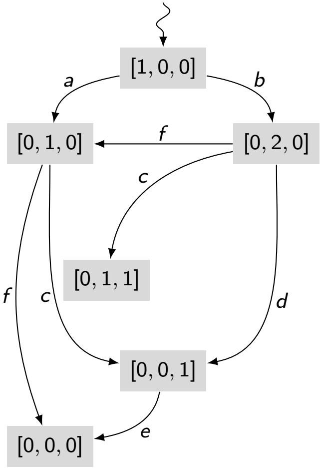


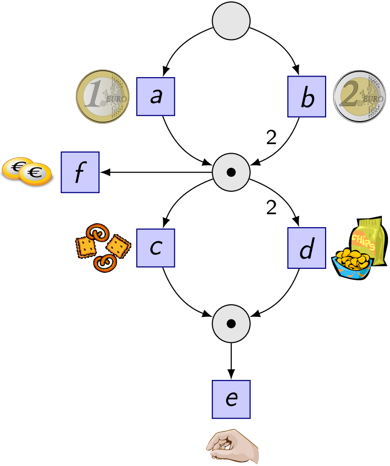
[1, 0, 0]
a
b
[0, 1, 0]
[0, 2, 0]
f
c
d
[0, 0, 1]
e
[0, 0, 0]

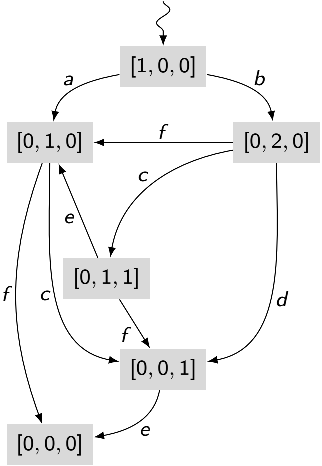

a
b
2
f
2
c
d
e

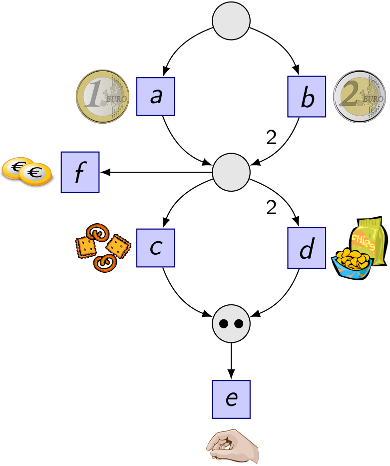

[1, 0, 0]
a
b
[0, 1, 0]
f
[0, 2, 0]
f
c
d
[0, 0, 1]
e
[0, 0, 0]

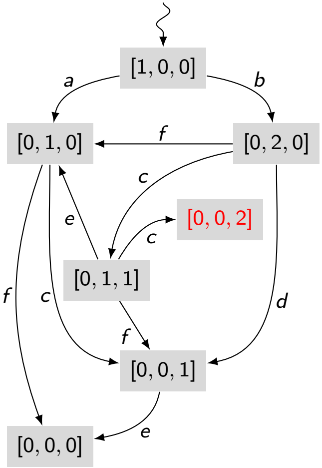

a
b
2
f
c
d
2
e
EURO
EURO
CHIPS

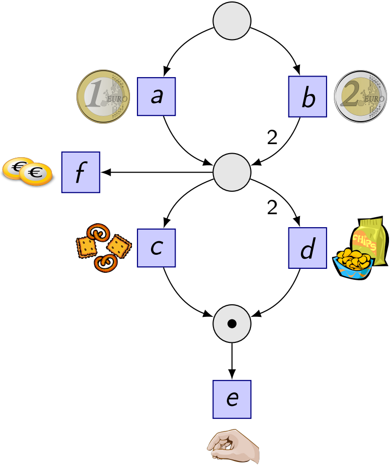

[1, 0, 0]
a
b
[0, 1, 0]
f
[0, 2, 0]
c
[0, 1, 1]
f
c
d
[0, 0, 1]
e
[0, 0, 0]

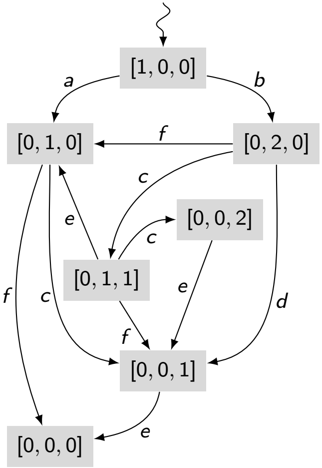

a
b
2
f
2
c
d
e


[1, 0, 0]
a
b
[0, 1, 0]
f
[0, 2, 0]
c
e
[0, 1, 1]
f
c
f
d
[0, 0, 1]
e
[0, 0, 0]


a
b
2
f
2
c
d
e


[1, 0, 0]
a
b
[0, 1, 0]
f
[0, 2, 0]
c
e
[0, 0, 2]
c
[0, 1, 1]
f
c
d
f
[0, 0, 1]
e
[0, 0, 0]

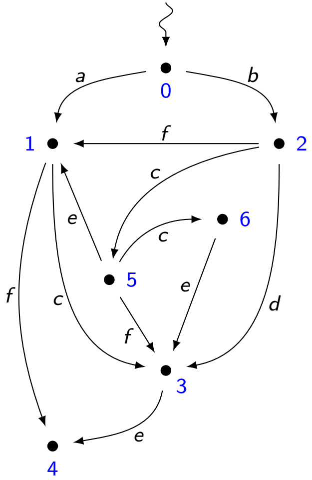

a
b
f
c
d
e
2
2
EURO
EURO
CHIPS

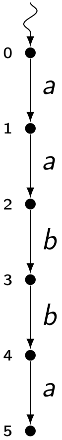

[1, 0, 0]
[0, 1, 0]
[0, 2, 0]
[0, 0, 2]
[0, 1, 1]
[0, 0, 1]
[0, 0, 0]
a
b
f
c
e
c
e
f
c
f
d
e

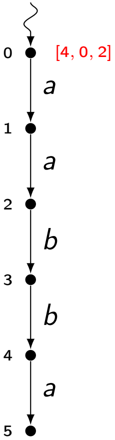
Labeled Transition System (LTS)

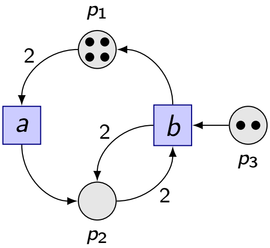
A tuple $TS = (S, T, \to, s_0)$, where:

- S – finite set of states
- T – finite set of letters (labels)
- $\to$ – finite set of edges
  $\to \subseteq (S \times T \times S)$
- $s_0 \in S$ – initial state

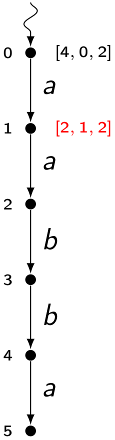


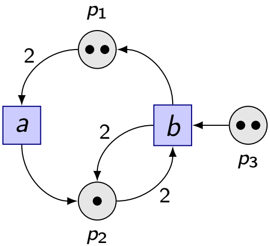

0
a
1
a
2
b
3
b
4
a
5



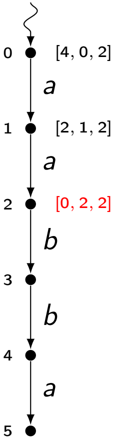


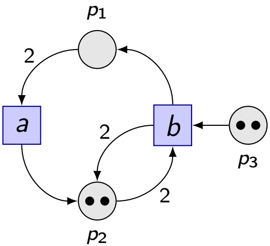
Solvable



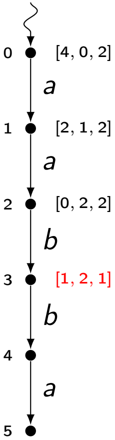


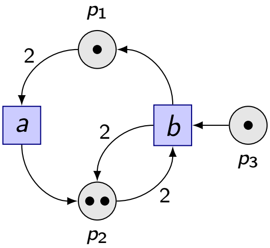
Solvable



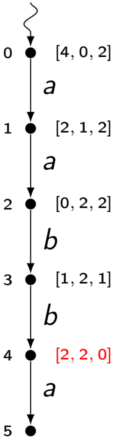


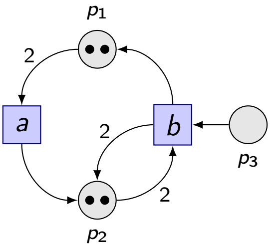
Solvable



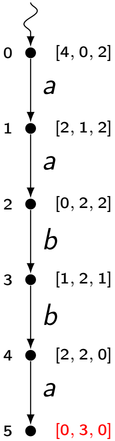


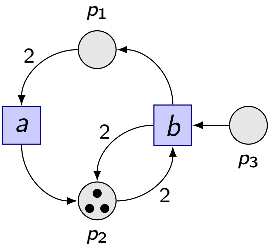
Solvable



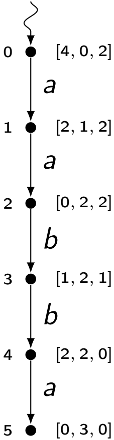


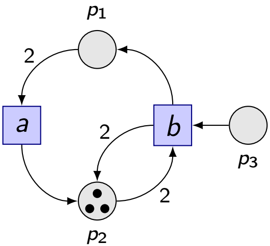
Solvable

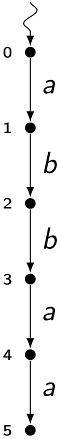




Solvable

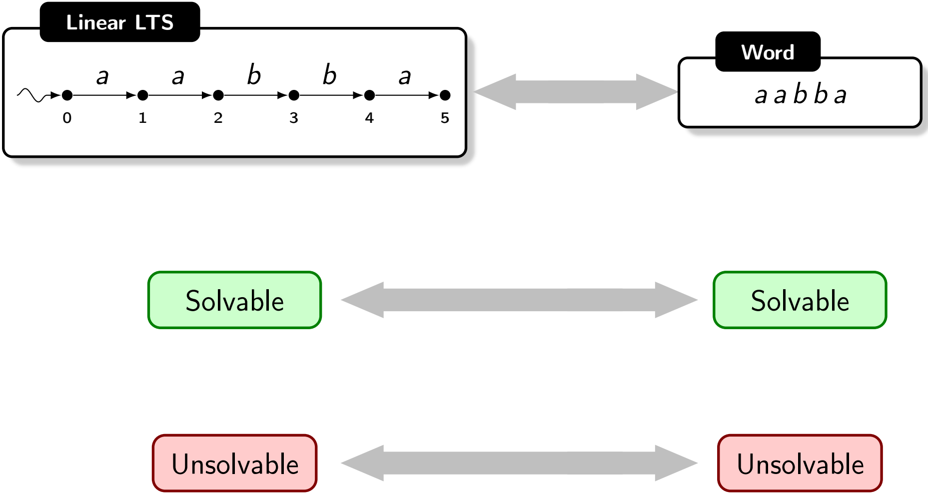




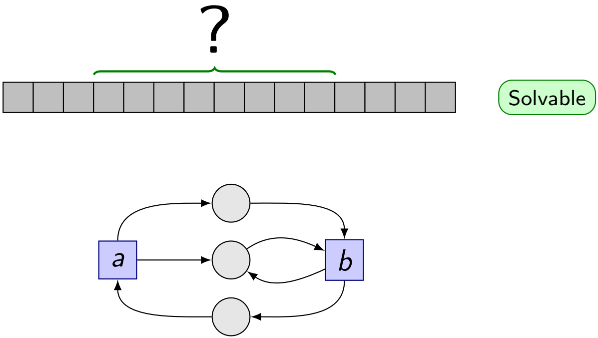


Solvable

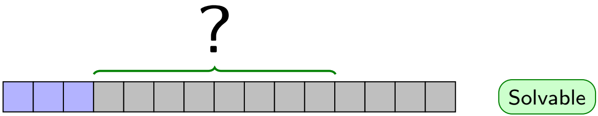
Unsolvable

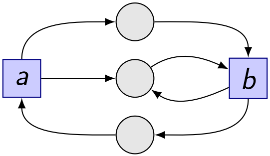

Linear LTS
a a b b a
0 1 2 3 4 5
Word
a a b b a
Solvable
Solvable
Unsolvable
Unsolvable

If **v** is solvable then all its factors are also solvable.

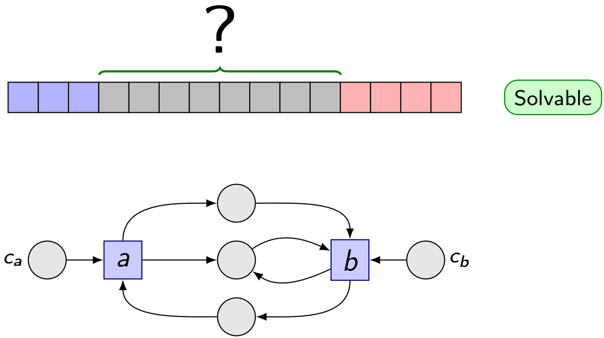


If **v** is solvable then all its factors are also solvable.

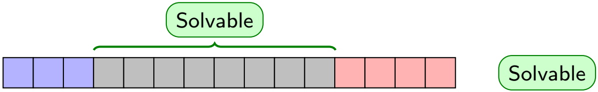


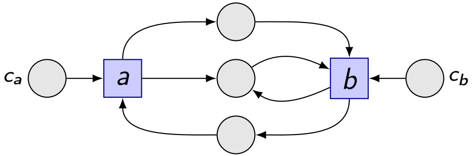


If **v** is solvable then all its factors are also solvable.



If **v** is solvable then all its factors are also solvable.

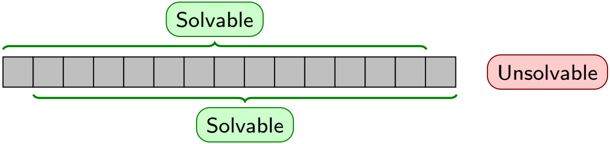




If $\mathbf{v}$ is unsolvable then $\mathbf{u} \cdot \mathbf{v} \cdot \mathbf{w}$ is unsolvable $(u, w \in \Sigma^*)$

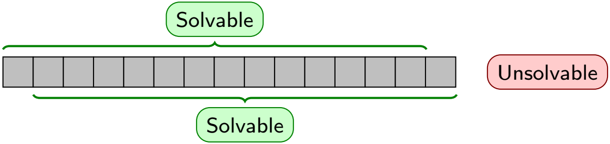
If **v** is unsolvable then $\mathbf{u} \cdot \mathbf{v} \cdot \mathbf{w}$ is unsolvable $(u, w \in \Sigma^*)$

Minimal Unsolvable Word (*muw*)



If **v** is unsolvable then $\mathbf{u} \cdot \mathbf{v} \cdot \mathbf{w}$ is unsolvable $(u, w \in \Sigma^*)$

Minimal Unsolvable Word (*muw*)



Problem

Characterize the language of all minimal unsolvable **binary** words.

Sufficient condition for unsolvability

If a word $w \in \{a, b\}^*$ contains a factor of the form

$(\, a\, b\, \alpha\,)\, b^*\, (\, b\, a\, \alpha\,)^+\, a$ or $(\, b\, a\, \alpha\,)\, a^*\, (\, a\, b\, \alpha\,)^+\, b$

where $\alpha \in \{a,\ b\}^*$, then w is **unsolvable**.

Sufficient condition for unsolvability

If a word $w \in \{a, b\}^*$ contains a factor of the form

$$(a\,b\,\alpha)\,b^*\,(b\,a\,\alpha)^+\,a \quad \text{or} \quad (b\,a\,\alpha)\,a^*\,(a\,b\,\alpha)^+\,b$$

where $\alpha \in \{a, b\}^*$, then w is **unsolvable**.

! If **aw** and **wb** are solvable then **awb** is solvable.

Sufficient condition for unsolvability

If a word $w \in \{a, b\}^*$ contains a factor of the form

$$(a\, b\, \alpha)\, b^*\, (b\, a\, \alpha)^+\, a \quad \text{or} \quad (b\, a\, \alpha)\, a^*\, (a\, b\, \alpha)^+\, b$$

where $\alpha \in \{a, b\}^*$, then w is **unsolvable**.

If **aw** and **wb** are solvable then **awb** is solvable.

If **aw** is solvable then **aaw** is solvable.

Sufficient condition for unsolvability

If a word $w \in \{a, b\}^*$ contains a factor of the form

$$(a\,b\,\alpha)\,b^*\,(b\,a\,\alpha)^+\,a \quad \text{or} \quad (b\,a\,\alpha)\,a^*\,(a\,b\,\alpha)^+\,b$$

where $\alpha \in \{a, b\}^*$, then w is **unsolvable**.

! If **aw** and **wb** are solvable then **awb** is solvable.

! If **aw** is solvable then **aaw** is solvable.

! If **awa** is minimal unsolvable then **w** does not contain either **aa** or **bb** as a factor.

Sufficient condition for unsolvability

If a word $w \in \{a, b\}^*$ contains a factor of the form

$(\, a\, b\, \alpha\,)\, b^*\, (\, b\, a\, \alpha\,)^+\, a$ or $(\, b\, a\, \alpha\,)\, a^*\, (\, a\, b\, \alpha\,)^+\, b$

where $\alpha \in \{a,\ b\}^*$, then w is **unsolvable**.

! If **aw** and **wb** are solvable then **awb** is solvable.

! If **aw** is solvable then **aaw** is solvable.

! If **awa** is minimal unsolvable then **w** does not contain either **aa** or **bb** as a factor.

! If $\mathbf{w} \in \mathbf{a}^*\mathbf{b}^+(\mathbf{ab}^+)^*(\mathbf{a}|\varepsilon)$ contains both $\mathbf{bab^ia}$ and $\mathbf{abb^ib}$ (with $\mathbf{x} \geq \mathbf{1}$) as factors, then **w** is not solvable.

Minimal Unsolvable Words

1. Non-extendable words ($\mathcal{NE}$)
2. Base Extendable words ($\mathcal{BE}$)
3. (Derivative) Extendable words ($\mathcal{E}$)
4. Extension operation (E)
5. Compression operation (C)

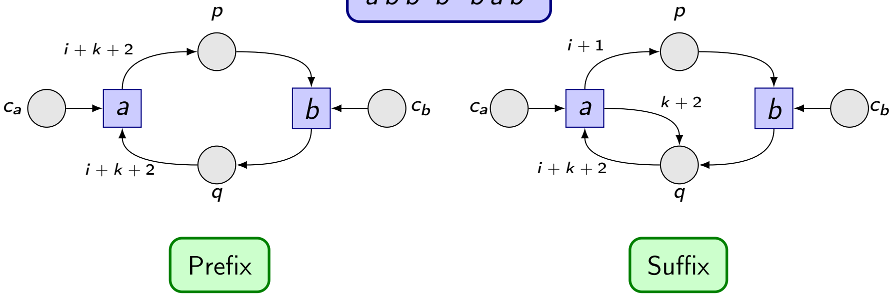
Non-extendable words ($\mathcal{NE}$)

A word $v \in \{a, b\}^*$ is called *non-extendable* if it is of the form

$$a\,b\,b^i b^k b\,a\,b^i a \quad \text{or} \quad b\,a\,a^i a^k a\,b\,a^i b$$

where $i \geq 0,\ k \geq 1$.

$a\,b\,b^i\,b^k\,b\,a\,b^i$



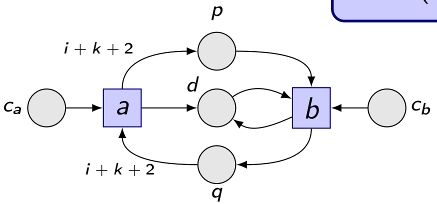
Base Extendable words ($\mathcal{BE}$)

A word $u \in \{a, b\}^*$ is called *base extendable* if it is of the form

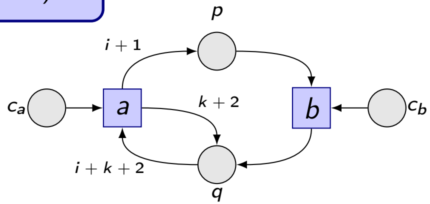
$$a\,b\,w\,(\,b\,a\,w\,)^k a \quad \text{or} \quad b\,a\,w\,(\,a\,b\,w\,)^k b$$

where $w = b^i$ or $w = a^i$, $i > 0$, $k \geq 1$.

$$a\,b\,b^i\,(\,b\,a\,b^i\,)^k\,a$$



Prefix



Suffix

Extension operation

$$E(a\,w\,a) = \bigcup_{i=1}^{\infty} \left\{ a\,b\,M_{a,i}(w)\,a^{i+1},\ \ a\,M_{b,i}(w\,a) \right\},$$

$$E(b\,w\,b) = \bigcup_{i=1}^{\infty} \left\{ b\,a\,M_{b,i}(w)\,b^{i+1},\ \ b\,M_{a,i}(w\,b) \right\},$$

$$M_{a,i} = \begin{cases} a \mapsto a^{i+1}b \\ b \mapsto a^{i}b \end{cases} \qquad M_{b,i} = \begin{cases} a \mapsto b^{i}a \\ b \mapsto b^{i+1}a \end{cases}$$

(Derivative) Extendable words ($\mathcal{E}$)

1. If $w \in E(v)$ and v is base extendable, then w is extendable.
2. If $w \in E(v)$ and v is extendable, then w is extendable.
3. There are no other extendable words.

Non-extendable words are **minimal unsolvable**.

Base extendable words are **minimal unsolvable**.

Extensions of extendable words are **minimal unsolvable**.

Extensions of non-extendable words are **unsolvable** but not **minimal unsolvable**.

Theorem

Let w be a minimal unsolvable binary word. Then we have the following exclusive alternatives:

- w is a non-extendable word ($w \in \mathcal{NE}$),
- w is a base extendable word ($w \in \mathcal{BE}$),
- w is an extendable word ($w \in \mathcal{E}$).

Compression operation

$$C(a\,b\,w\,a^{i+1}) \;=\; a\,M_{a,i}^{-1}(w)\,a \qquad C(b\,a\,wb^{i+1}) \;=\; b\,M_{b,i}^{-1}(w)\,b$$

$$C(a\,w\,b\,a) \;=\; a\,M_{b,i}^{-1}(w\,b\,a) \qquad C(b\,w\,a\,b) \;=\; b\,M_{a,i}^{-1}(w\,a\,b)$$

$$M_{a,i}^{-1} = \begin{cases} a^{i+1}b & \mapsto & a \\ a^{i}b & \mapsto & b \end{cases} \qquad M_{b,i}^{-1} = \begin{cases} b^{i}a & \mapsto & a \\ b^{i+1}a & \mapsto & b \end{cases}$$

Lemma

1. If $v \in \mathcal{BE} \cup \mathcal{E}$ and $u \in E(v)$, then $C(u) = v$;
2. If $u \in \mathcal{C}$ and $v = C(u)$, then $u \in E(v)$.

Algorithm – Minimal Unsolvability of v

if v matches pattern (Ia) or (Ib) **then**
    return MINIMAL UNSOLVABLE;
while *true* **do**
    if v matches pattern (IIa) or (IIb) **then**
        return MINIMAL UNSOLVABLE;
    if v compressible **then**
        $v \leftarrow C(v)$;
    else
        return NOT MINIMAL UNSOLVABLE;

Non-extendable

(Ia) $a\,b^x a\,b^y a$

(Ib) $b\,a^x b\,a^y b$

$x > y + 2, \; y \geq 0$

Base Extendable

(IIa) $a\,b\,w\,(\,b\,a\,w\,)^k\,a$

(IIb) $b\,a\,w\,(a\,b\,w\,)^k\,b$

$w = b^i$ or $w = a^i$,
$i > 0, \; k \geq 1$

$$b\,a\,a\,a\,b\,a\,a\,a\,b\,a\,a\,a\,b\,a\,a\,b\,a\,a\,a\,b\,a\,a\,b$$

Non-extendable

(Ia) $a\,b^x a\,b^y a$

(Ib) $b\,a^x b\,a^y b$

$x > y + 2,\ y \geq 0$

Base Extendable

(IIa) $a\,b\,w\,(\,b\,a\,w\,)^k\,a$

(IIb) $b\,a\,w\,(a\,b\,w\,)^k\,b$

$w = b^i$ or $w = a^i$, $i > 0,\ k \geq 1$

$$M_{a,i}^{-1} = \begin{cases} a^{i+1}b & \mapsto & a \\ a^i b & \mapsto & b \end{cases}$$

$$M_{b,i}^{-1} = \begin{cases} b^i a & \mapsto & a \\ b^{i+1} a & \mapsto & b \end{cases}$$

$$\mathrm{b\,a}\,\alpha\,\mathrm{a}^*\,(\,\mathrm{a\,b}\,\alpha\,)^+\,\mathrm{b}$$

$$\mathrm{b}\,\underbrace{\mathrm{a\,a\,a\,b\,a\,a\,a\,b\,a}}_{\alpha}\,\mathrm{a\,a\,b}\,\underbrace{\mathrm{a\,a\,b\,a\,a\,a\,b\,a\,a}}_{\alpha}\,\mathrm{b}$$

Non-extendable

(Ia) $a\,b^x a\,b^y a$

(Ib) $b\,a^x b\,a^y b$

$x > y + 2,\ y \geq 0$

Base Extendable

(IIa) $a\,b\,w\,(\,b\,a\,w\,)^k\,a$

(IIb) $b\,a\,w\,(\,a\,b\,w\,)^k\,b$

$w = b^i$ or $w = a^i$, $i > 0,\ k \geq 1$

$$M_{a,i}^{-1} = \begin{cases} a^{i+1}b & \mapsto & a \\ a^i b & \mapsto & b \end{cases}$$

$$M_{b,i}^{-1} = \begin{cases} b^i a & \mapsto & a \\ b^{i+1} a & \mapsto & b \end{cases}$$

Unsolvable $\mathrm{b\,a\,\alpha\,a^{*}\,(\,a\,b\,\alpha\,)^{+}\,b}$

$$\mathrm{b\,\underbrace{a\,a\,a\,b\,a\,a\,a\,b\,a\,a\,a}_{\alpha}\,b\,\underbrace{a\,a\,b\,a\,a\,a\,b\,a\,a}_{\alpha}\,b}$$

Non-extendable

(Ia) $a\,b^{x}a\,b^{y}a$

(Ib) $b\,a^{x}b\,a^{y}b$

$x > y+2,\ y \geq 0$

Base Extendable

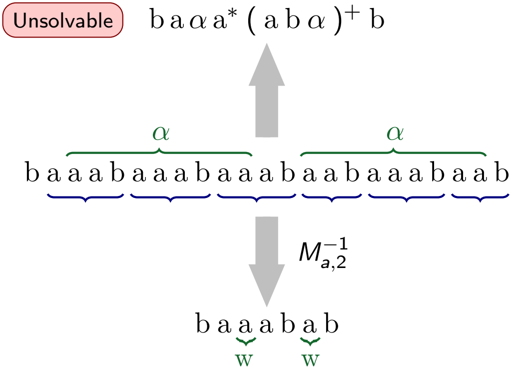
(IIa) $a\,b\,w\,(\,b\,a\,w\,)^{k}\,a$

(IIb) $b\,a\,w\,(a\,b\,w\,)^{k}\,b$

$w = b^{i}$ or $w = a^{i}$, $i > 0,\ k \geq 1$

$$M_{a,i}^{-1} = \begin{cases} a^{i+1}b & \mapsto & a \\ a^{i}b & \mapsto & b \end{cases}$$

$$M_{b,i}^{-1} = \begin{cases} b^{i}a & \mapsto & a \\ b^{i+1}a & \mapsto & b \end{cases}$$



Non-extendable

(Ia) $a\,b^x a\,b^y a$

(Ib) $b\,a^x b\,a^y b$

$x > y + 2,\ y \geq 0$

Base Extendable

(IIa) $a\,b\,w\,(\,b\,a\,w\,)^k\,a$

(IIb) $b\,a\,w\,(a\,b\,w\,)^k\,b$

$w = b^i$ or $w = a^i$, $i > 0,\ k \geq 1$

$$M_{a,i}^{-1} = \begin{cases} a^{i+1}b & \mapsto & a \\ a^i b & \mapsto & b \end{cases}$$

$$M_{b,i}^{-1} = \begin{cases} b^i a & \mapsto & a \\ b^{i+1} a & \mapsto & b \end{cases}$$

Unsolvable $\mathrm{b\,a\,\alpha\,a^*\,(\,a\,b\,\alpha\,)^+\,b}$

$$\underbrace{\mathrm{b\,a}\overbrace{\mathrm{a\,a\,b\,a\,a\,a\,b\,a\,a\,a}}^{\alpha}\mathrm{b}\overbrace{\mathrm{a\,a\,b\,a\,a\,a\,b\,a\,a}}^{\alpha}\mathrm{b}}$$

b a a a b a a a b a a a b a a b a a a b a a b

$M_{a,2}^{-1}$

Minimal unsolvable $\mathrm{b\,a\,\underbrace{a}_{w}\,a\,b\,\underbrace{a}_{w}\,b}$

Non-extendable

(Ia) $a\,b^x a\,b^y a$

(Ib) $b\,a^x b\,a^y b$

$x > y + 2,\ y \geq 0$

Base Extendable

(IIa) $a\,b\,w\,(\,b\,a\,w\,)^k\,a$

(IIb) $b\,a\,w\,(a\,b\,w\,)^k\,b$

$w = b^i$ or $w = a^i$,

$i > 0,\ k \geq 1$

$$M_{a,i}^{-1} = \begin{cases} a^{i+1}b & \mapsto & a \\ a^i b & \mapsto & b \end{cases}$$

$$M_{b,i}^{-1} = \begin{cases} b^i a & \mapsto & a \\ b^{i+1} a & \mapsto & b \end{cases}$$

Unsolvable $\mathrm{b\,a\,\alpha\,a^*\,(\,a\,b\,\alpha\,)^+\,b}$

$$\mathrm{b\,a\,\underbrace{\overbrace{a\,a\,b\,a\,a\,a\,b}^{\alpha}}\,\ldots}$$

b a a a b a a a b a a a b a a b a a a b a a b

Minimal unsolvable $\mathrm{b\,a\,\underbrace{a}_{w}\,a\,b\,\underbrace{a}_{w}\,b}$

---

a b b b a b a

$\downarrow M_{b,2}$

a b b b a b b b a b b b a b b a b b b a b b a

Non-extendable

(Ia) $a\,b^x a\,b^y a$

(Ib) $b\,a^x b\,a^y b$

$x > y + 2, \ y \geq 0$

Base Extendable

(IIa) $a\,b\,w\,(\,b\,a\,w\,)^k\,a$

(IIb) $b\,a\,w\,(a\,b\,w\,)^k\,b$

$w = b^i$ or $w = a^i$,
$i > 0, \ k \geq 1$

$$M_{a,i} = \begin{cases} a \mapsto a^{i+1}b \\ b \mapsto a^i b \end{cases}$$

$$M_{b,i} = \begin{cases} a \mapsto b^i a \\ b \mapsto b^{i+1} a \end{cases}$$

Necessary and sufficient condition for unsolvability

A word $w \in \{a, b\}^*$ is **unsolvable** if and only if it contains a factor of the form

$$(a\,b\,\alpha)\,b^*\,(b\,a\,\alpha)^+\,a \qquad \text{or} \qquad (b\,a\,\alpha)\,a^*\,(a\,b\,\alpha)^+\,b$$

where $w \in \{a, b\}^*$.

Non-extendable words

(Ia) $a\,b^x a\,b^y a$

(Ib) $b\,a^x b\,a^y b$

$x > y + 2,\ y \geq 0$

Extendable words

(IIa) $a\,b\,w\,(\,b\,a\,w\,)^k\,a$

(IIb) $b\,a\,w\,(a\,b\,w\,)^k\,b$

$k \geq 1,\ w \in \{a, b\}^*$

Algorithm – Unsolvability of v

if v contains pattern (Ia) or (Ib) **then**
  return UNSOLVABLE; } $O(n)$

foreach $v[i] \neq v[i+1]$ $(i = 1..n)$ **do**
  swap($v[i] \leftrightarrow v[i+1]$);
  compute border array for $v[i..n]$; } $O(n)$
  if $v[i..j] = w^k$ *and* $v[j+1] = v[i+1]$ **then**
    return UNSOLVABLE;
  swap($v[i] \leftrightarrow v[i+1]$);
} $O(n^2)$

return SOLVABLE;

Non-extendable

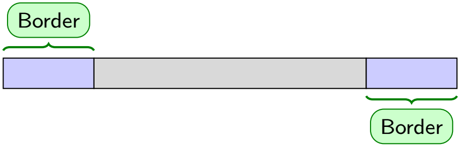
(Ia) $a\,b^x a\,b^y a$
(Ib) $b\,a^x b\,a^y b$

$x > y + 2, \;\; y \geq 0$

Extendable

(IIa) $a\,b\,w\,(\,b\,a\,w\,)^k\,a$
(IIb) $b\,a\,w\,(a\,b\,w\,)^k\,b$

$k \geq 1, \;\; w \in \{a,\, b\}^*$



Algorithm – Unsolvability of v

if v contains pattern (Ia) or (Ib) **then**
    return UNSOLVABLE; } $O(n)$

foreach $v[i] \neq v[i+1]$ $(i = 1..n)$ **do**
    swap($v[i] \leftrightarrow v[i+1]$);
    compute border array for $v[i..n]$; } $O(n)$
    if $v[i..j] = w^k$ *and* $v[j+1] = v[i+1]$ **then**
        return UNSOLVABLE;
    swap($v[i] \leftrightarrow v[i+1]$);
} $O(n^2)$

return SOLVABLE;

Non-extendable

(Ia) $a\,b^x a\,b^y a$

(Ib) $b\,a^x b\,a^y b$

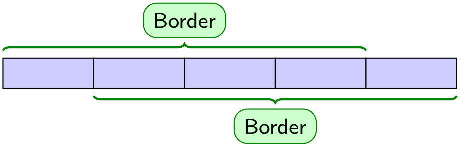
$x > y + 2, \ \ y \geq 0$

Extendable

(IIa) $a\,b\,w\,(\,b\,a\,w\,)^k\,a$

(IIb) $b\,a\,w\,(a\,b\,w\,)^k\,b$

$k \geq 1, \ \ w \in \{a, \ b\}^*$



Algorithm – Unsolvability of v

if v contains pattern (Ia) or (Ib) **then**
  return UNSOLVABLE;
} $O(n)$

foreach $v[i] \neq v[i+1]$ $(i = 1..n)$ **do**
  swap($v[i] \leftrightarrow v[i+1]$);
  compute border array for $v[i..n]$; } $O(n)$
  if $v[i..j] = w^k$ *and* $v[j+1] = v[i+1]$ **then**
    return UNSOLVABLE;
  swap($v[i] \leftrightarrow v[i+1]$);
} $O(n^2)$

return SOLVABLE;

Non-extendable

(Ia) $a\, b^x a\, b^y a$

(Ib) $b\, a^x b\, a^y b$

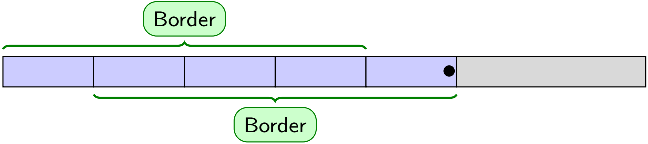
$x > y + 2, \ y \geq 0$

Extendable

(IIa) $a\, b\, w\, (\, b\, a\, w\,)^k\, a$

(IIb) $b\, a\, w\, (a\, b\, w\,)^k\, b$

$k \geq 1, \ w \in \{a, \, b\}^*$



Algorithm – Unsolvability of v

if v contains pattern (Ia) or (Ib) **then** — $O(n)$
  return UNSOLVABLE;

foreach $v[i] \neq v[i+1]$ $(i = 1..n)$ **do** — $O(n^2)$
  swap($v[i] \leftrightarrow v[i+1]$);
  compute border array for $v[i..n]$; — $O(n)$
  if $v[i..j] = w^k$ *and* $v[j+1] = v[i+1]$ **then**
    return UNSOLVABLE;
  swap($v[i] \leftrightarrow v[i+1]$);

return SOLVABLE;

Non-extendable

(Ia) $a\,b^x a\,b^y a$

(Ib) $b\,a^x b\,a^y b$

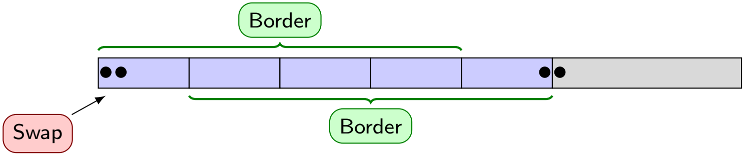
$x > y + 2, \; y \geq 0$

Extendable

(IIa) $a\,b\,w\,(b\,a\,w)^k\,a$

(IIb) $b\,a\,w\,(a\,b\,w)^k\,b$

$k \geq 1, \; w \in \{a, b\}^*$



Algorithm – Unsolvability of v

if v contains pattern (Ia) or (Ib) **then** } $O(n)$
  return UNSOLVABLE;

foreach $v[i] \neq v[i+1]$ $(i = 1..n)$ **do**
  swap($v[i] \leftrightarrow v[i+1]$);
  compute border array for $v[i..n]$; } $O(n)$
  if $v[i..j] = w^k$ *and* $v[j+1] = v[i+1]$ **then**
    return UNSOLVABLE;
  swap($v[i] \leftrightarrow v[i+1]$);
} $O(n^2)$

return SOLVABLE;

Non-extendable

(Ia) $a\,b^x a\,b^y a$

(Ib) $b\,a^x b\,a^y b$

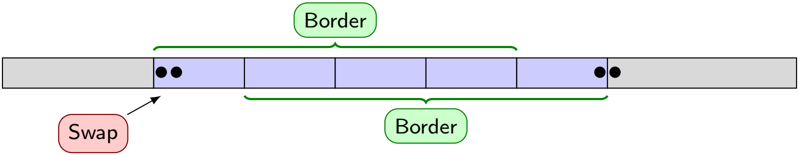
$x > y + 2, \; y \geq 0$

Extendable

(IIa) $a\,b\,w\,(b\,a\,w)^k\,a$

(IIb) $b\,a\,w\,(a\,b\,w)^k\,b$

$k \geq 1, \; w \in \{a, b\}^*$



Algorithm – Unsolvability of v

if v contains pattern (Ia) or (Ib) **then** } $O(n)$
    return UNSOLVABLE;

foreach $v[i] \neq v[i+1]$ $(i = 1..n)$ **do**
    swap($v[i] \leftrightarrow v[i+1]$);
    compute border array for $v[i..n]$; } $O(n)$
    if $v[i..j] = w^k$ *and* $v[j+1] = v[i+1]$ **then**
        return UNSOLVABLE;
    swap($v[i] \leftrightarrow v[i+1]$);
} $O(n^2)$

return SOLVABLE;

Non-extendable

(Ia) $a\,b^x a\,b^y a$
(Ib) $b\,a^x b\,a^y b$

$x > y + 2, \; y \geq 0$

Extendable

(IIa) $a\,b\,w\,(b\,a\,w)^k\,a$
(IIb) $b\,a\,w\,(a\,b\,w)^k\,b$

$k \geq 1, \; w \in \{a, b\}^*$

Conclusions

- Characterization of the language of Petri net unsolvable binary words.
- Method for checking **minimal unsolvability** for binary words.
- Efficient algorithm for testing **unsolvability** for binary words.

Conclusions

- Characterization of the language of Petri net unsolvable binary words.
- Method for checking **minimal unsolvability** for binary words.
- Efficient algorithm for testing **unsolvability** for binary words.

Future work

- Larger alphabets
- More complicated LTS's
- Other classes of nets
- Approximate solvability

Thank you

Dziękuję

Спасибо

Děkuji