

# The Gapped-Factor Tree

## PSC'06

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# Outline

Goal - Motivations

Overview

Preliminaries

Ukkonen suffix tree construction

$k$ -factor tree construction (Allali - Sagot)

Construction Algorithm

Construction

Complexity

Conclusion



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# Goal

- Indexation of *gapped-factors* :
  - A  $k$ -factor, a gap of length  $d$ , and a  $k'$ -factor
  - a  $(k, d, k')$ -gapped-factor

A T A T A G T T A G T ...  
0 1 2 3 4 5 6 7 8 9 ...



# Motivations

## Stringology

- Extensive use of  $k$ -factors ( $q$ -gram,  $k$ -mer)
- Gapped-factors for sets of  $k$ -factors
- Indexation structure : interesting application of the suffix tree

## Bioinformatics

- Motif inference
- Binding site detection



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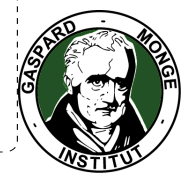
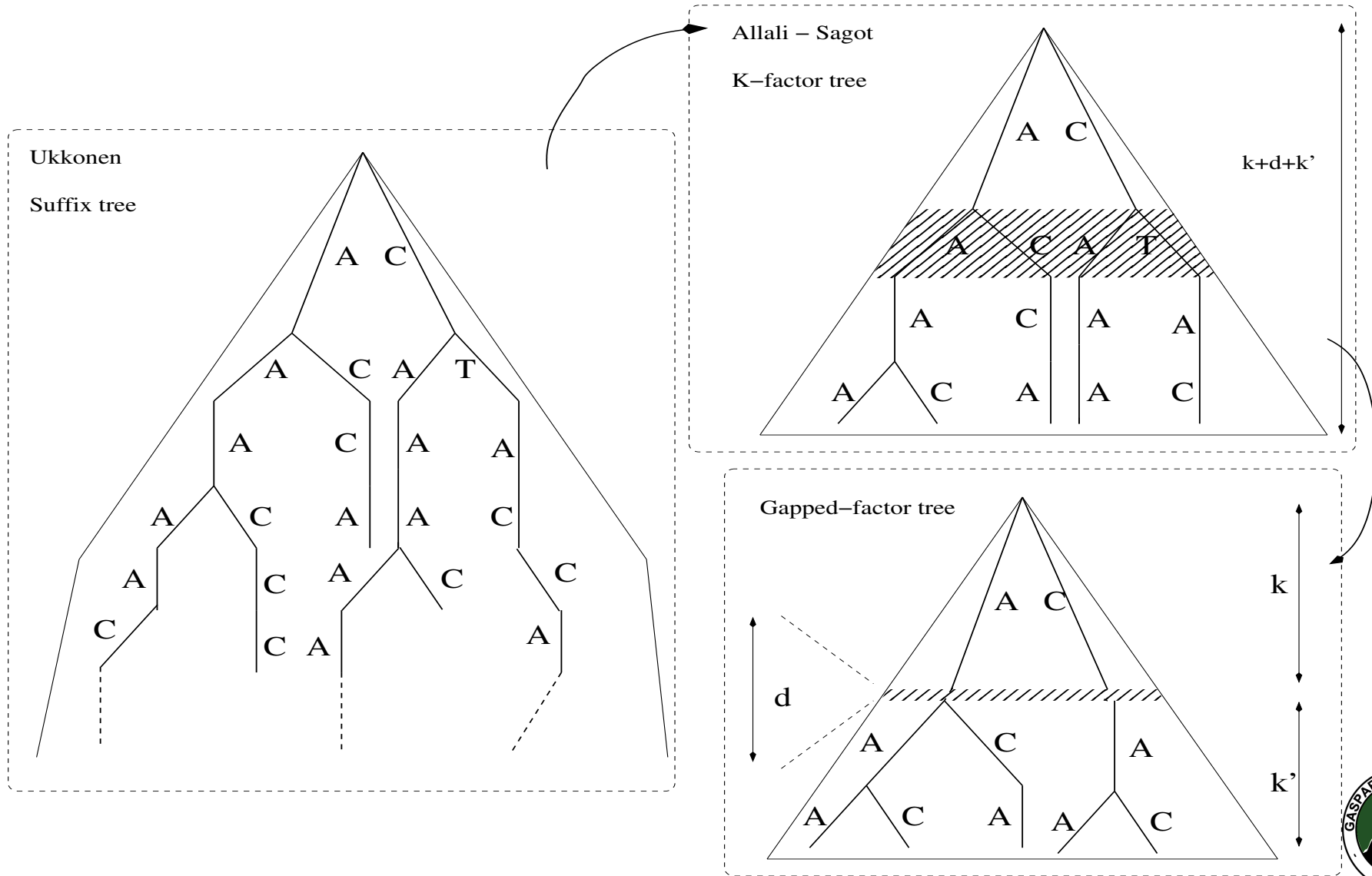
Construction Algorithm

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# Overview



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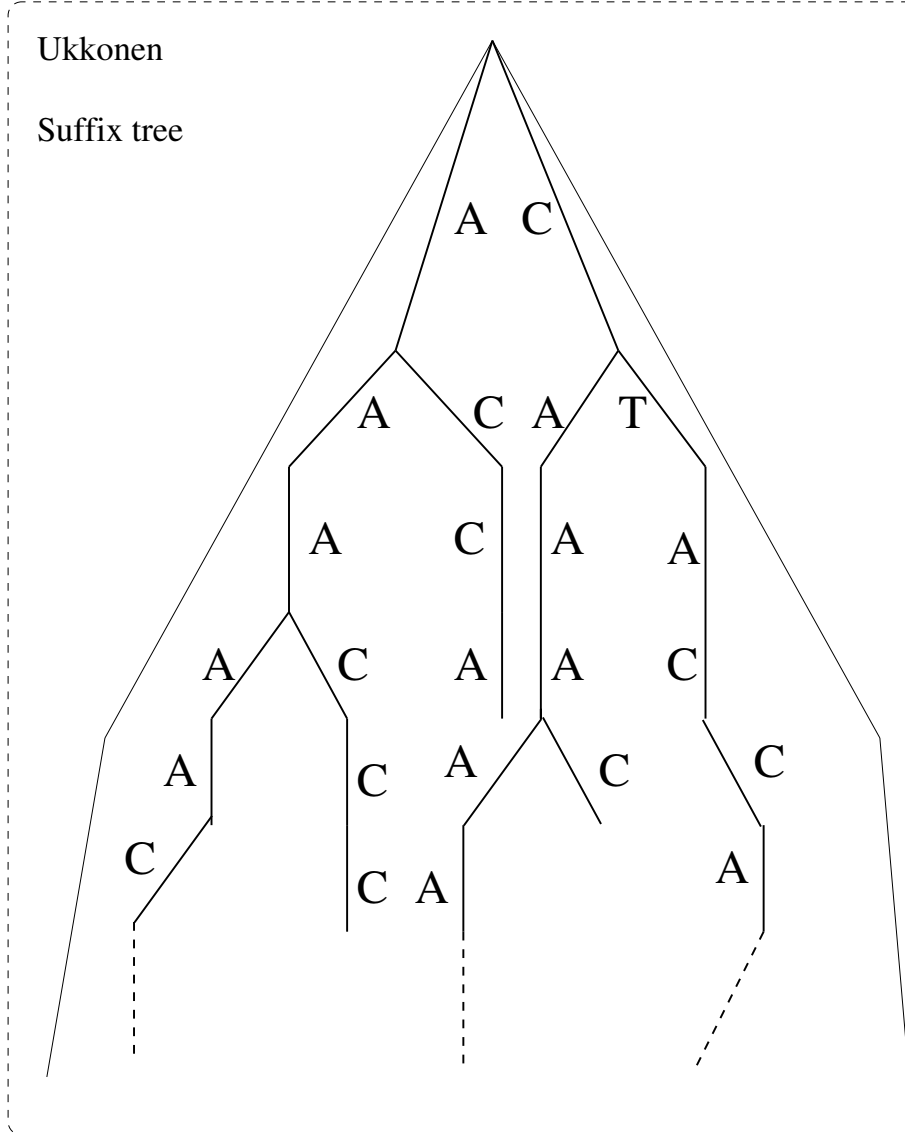




# Overview

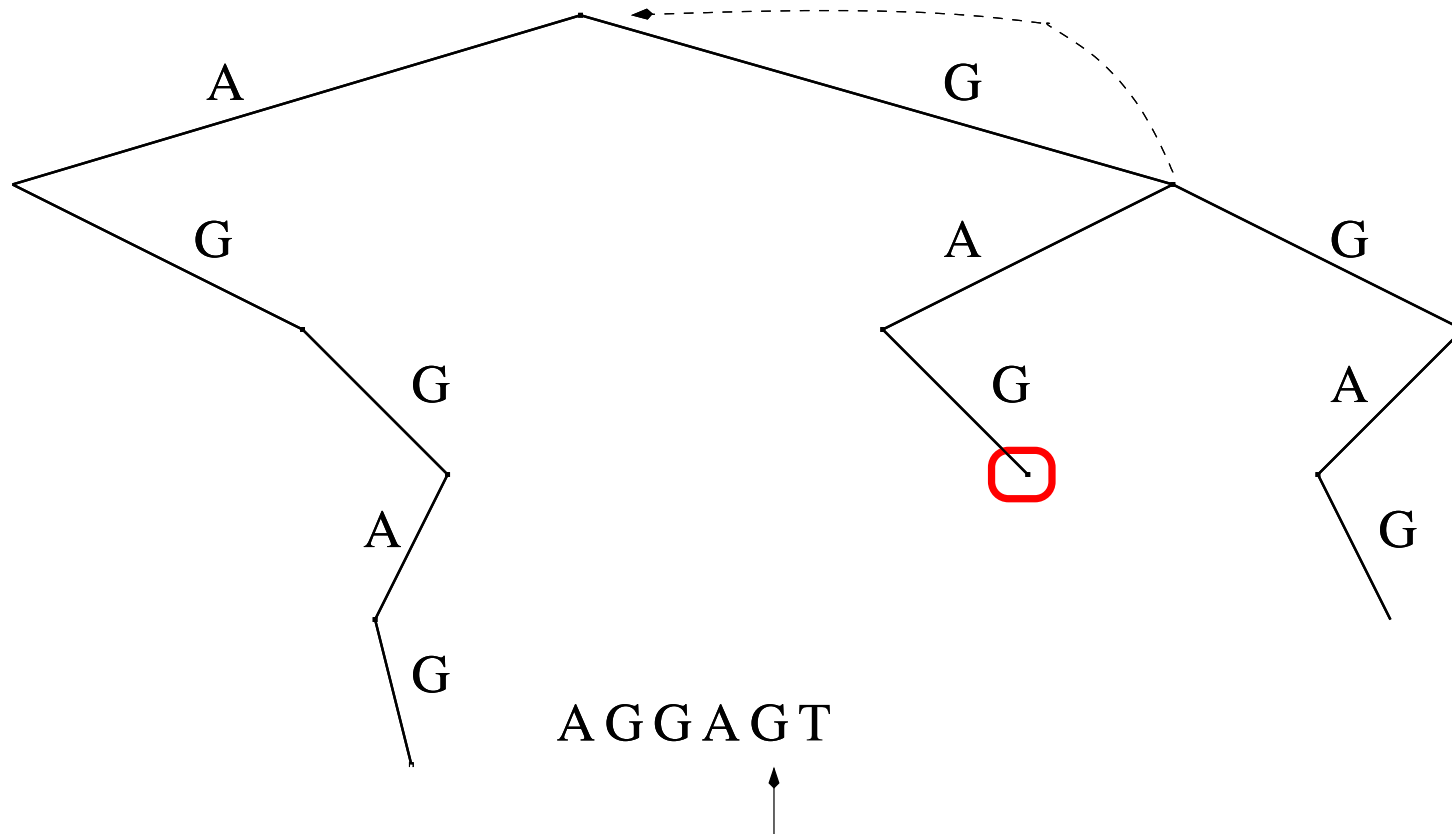
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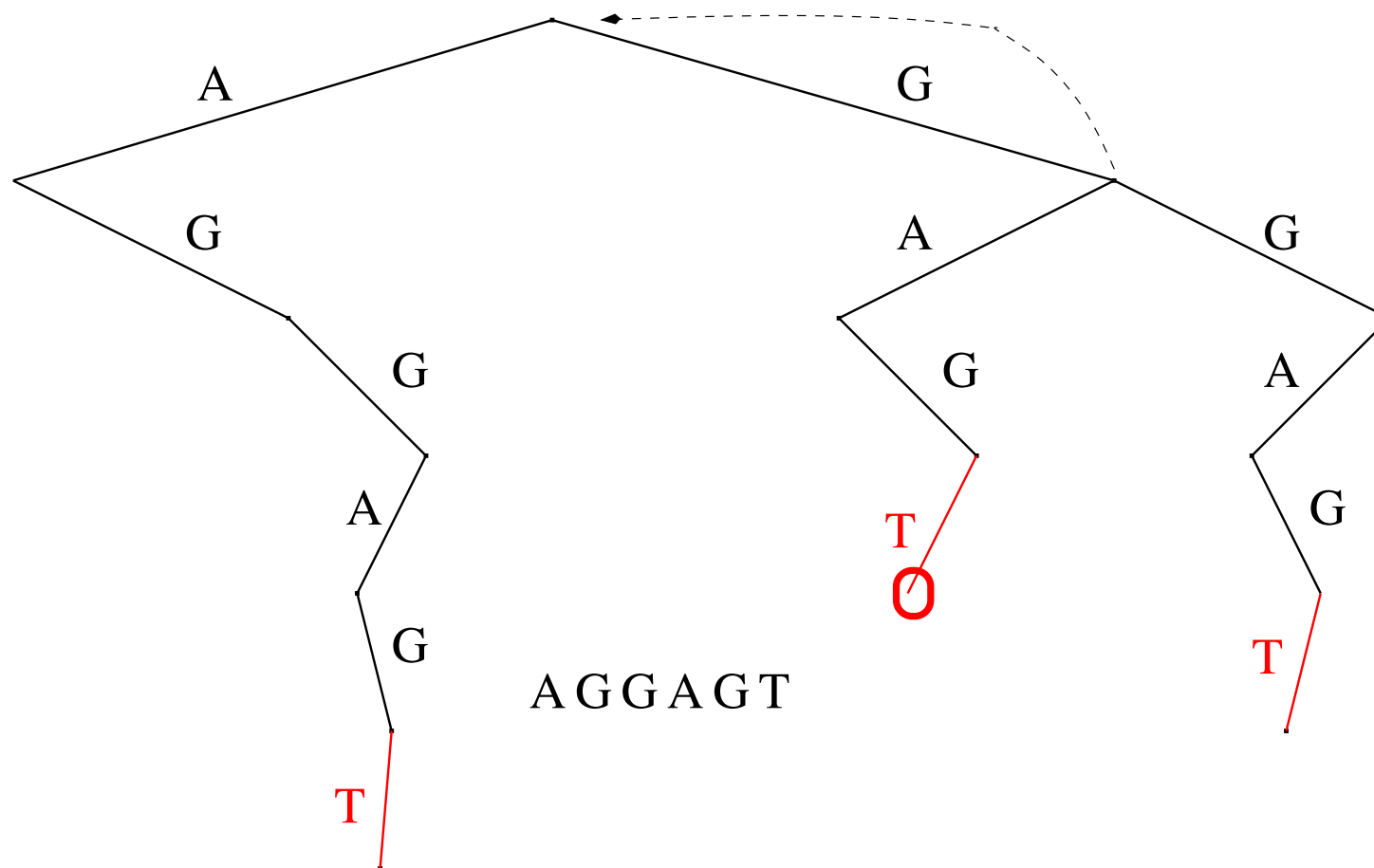
# Suffix tree construction

Suffix tree for *AGGAG*, *last\_leaf* is marked

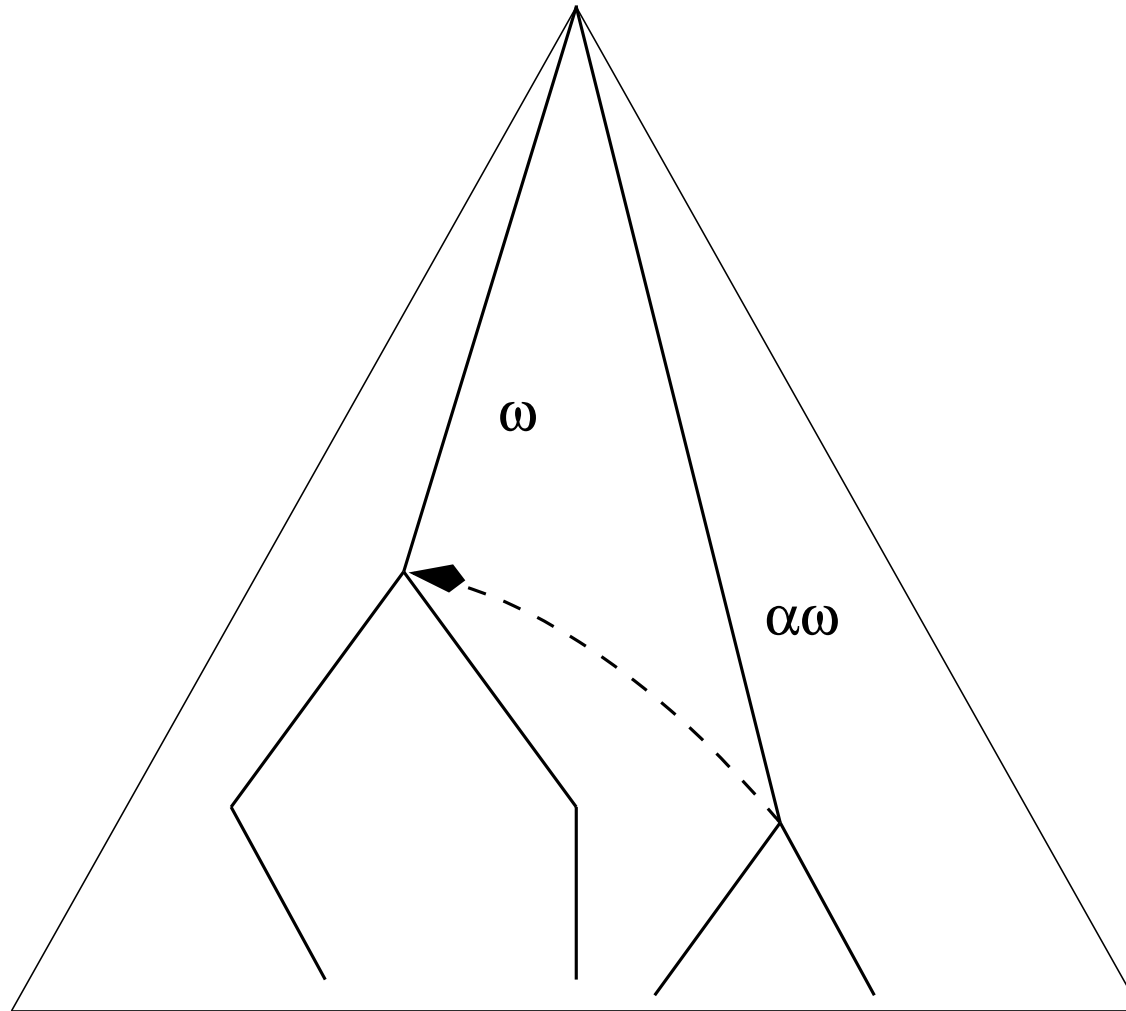


## Suffix tree construction

Adding  $T$  to  $AGGAG$ , **implicit extension** of all leaves  
[ $Start - End$ ]  $\Rightarrow end$  : global variable

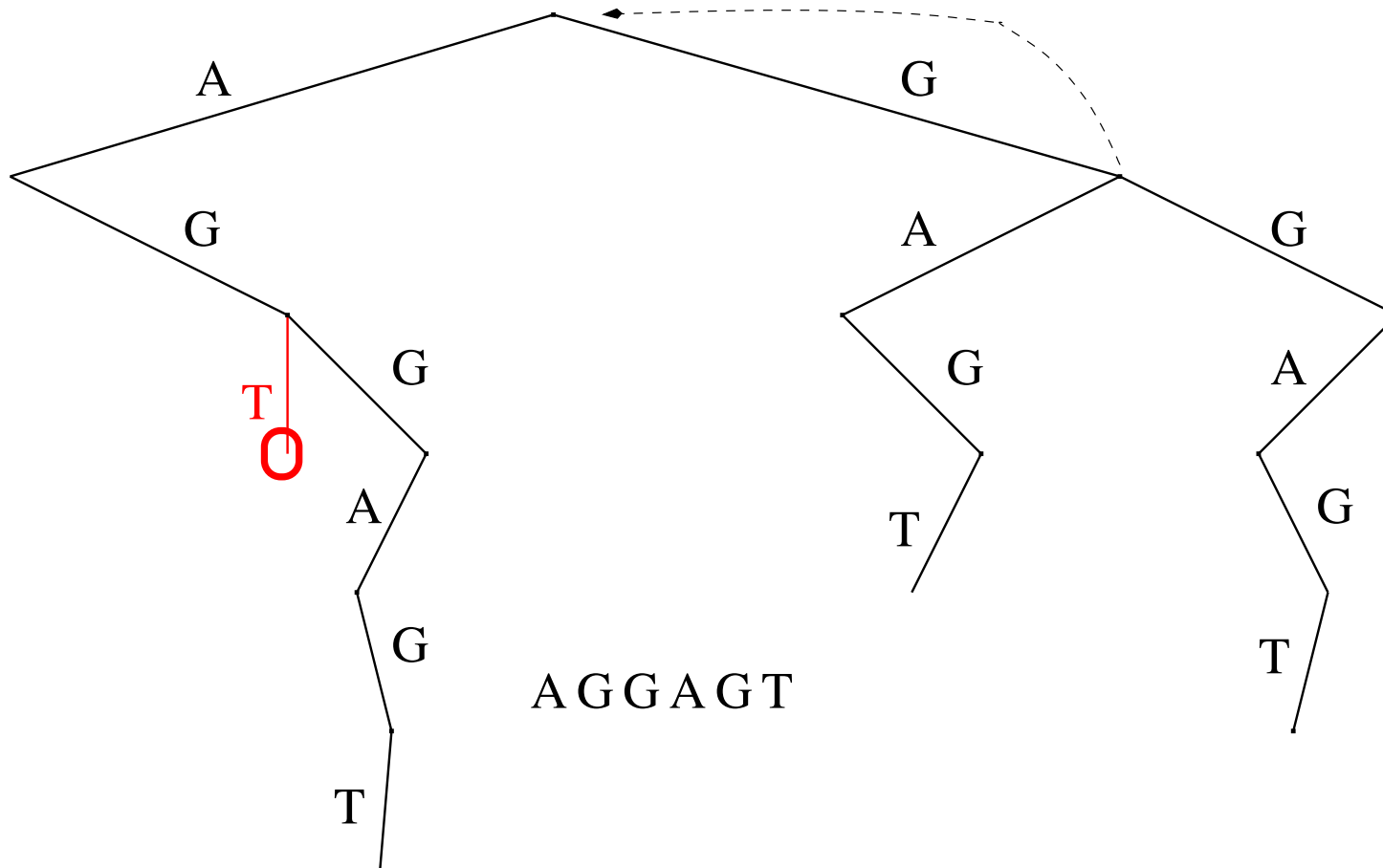


# Suffix tree construction



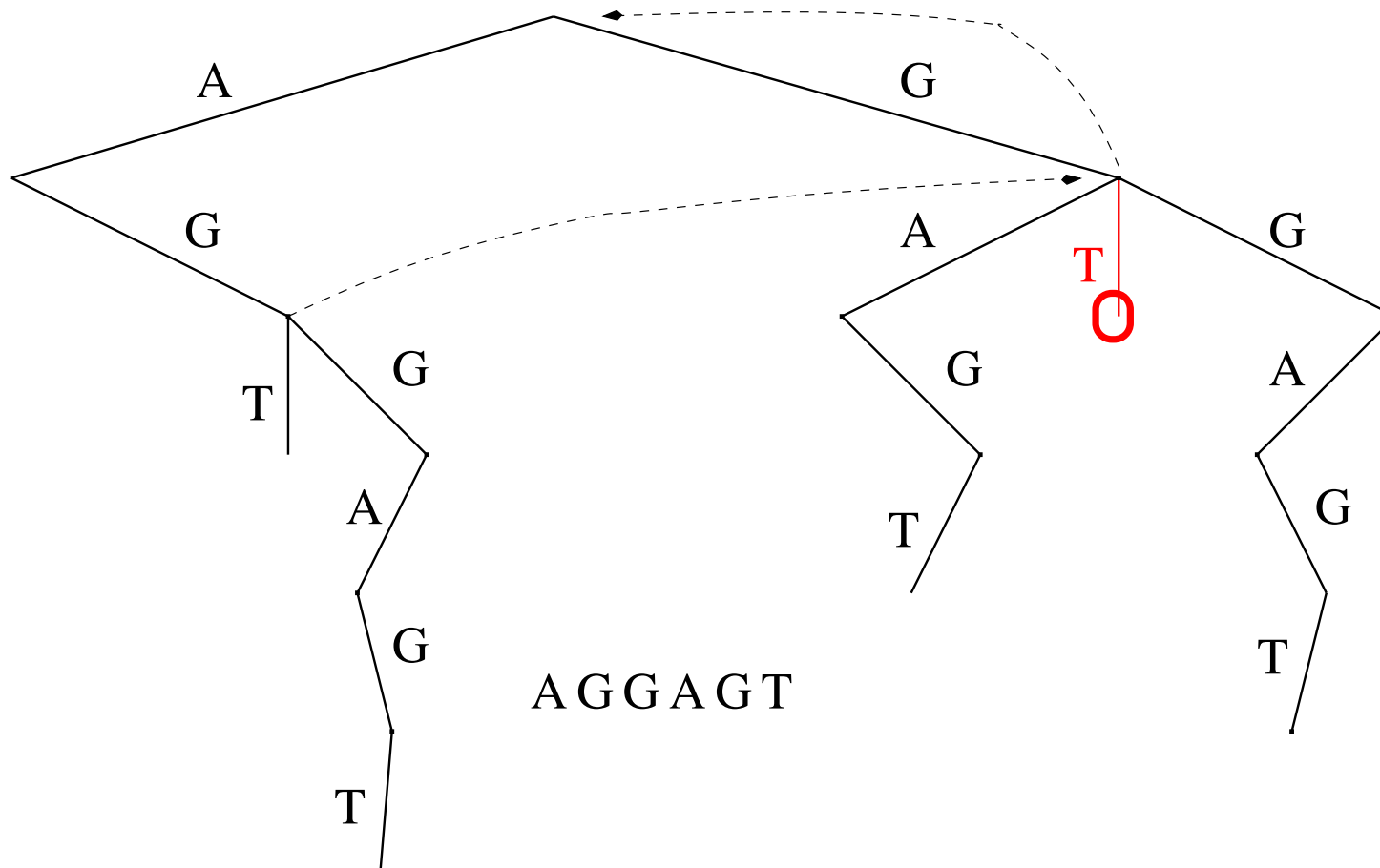
# Suffix tree construction

Adding  $T$  to  $AGGAG$ , fast insertion of  $AGT$  from the root



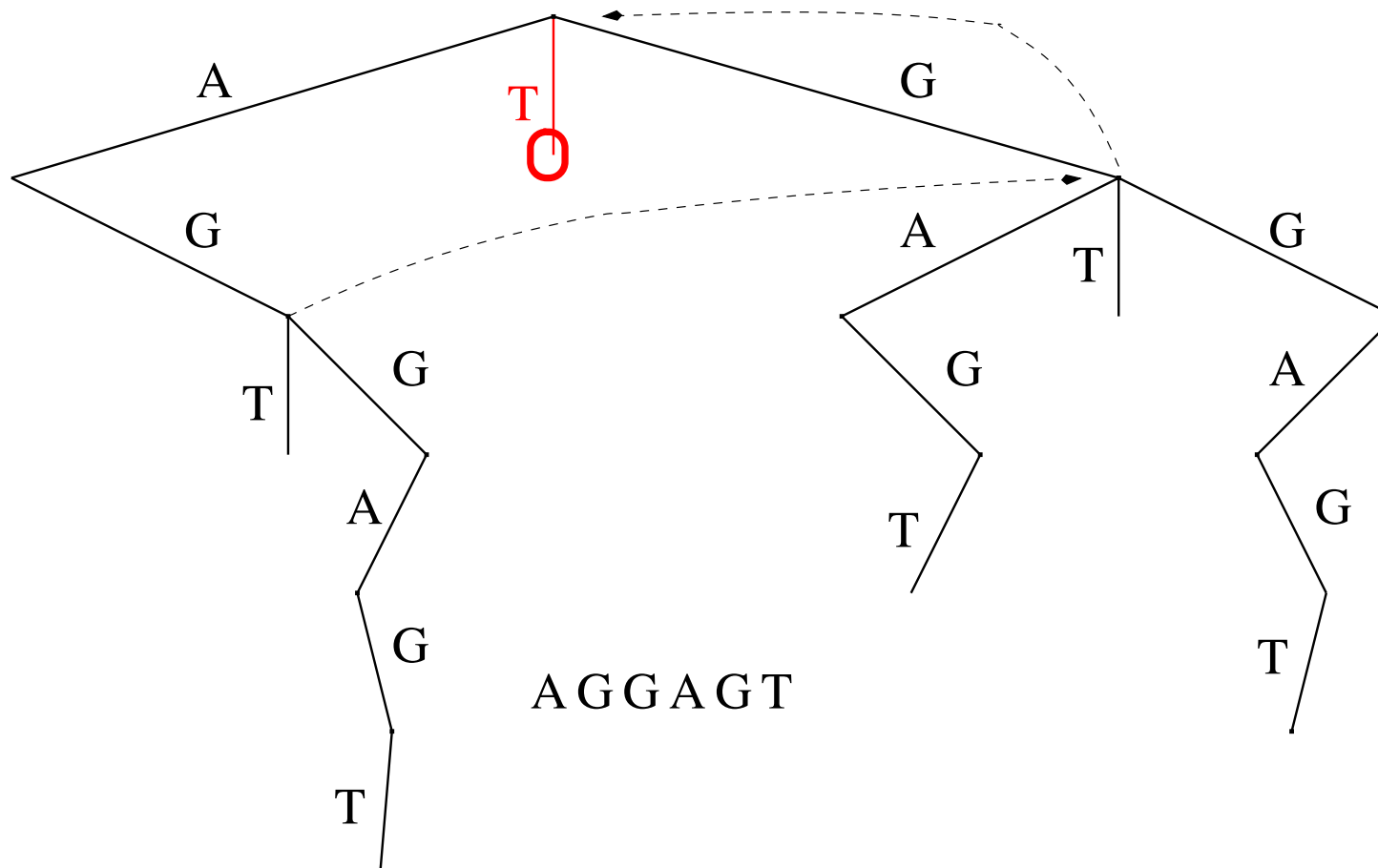
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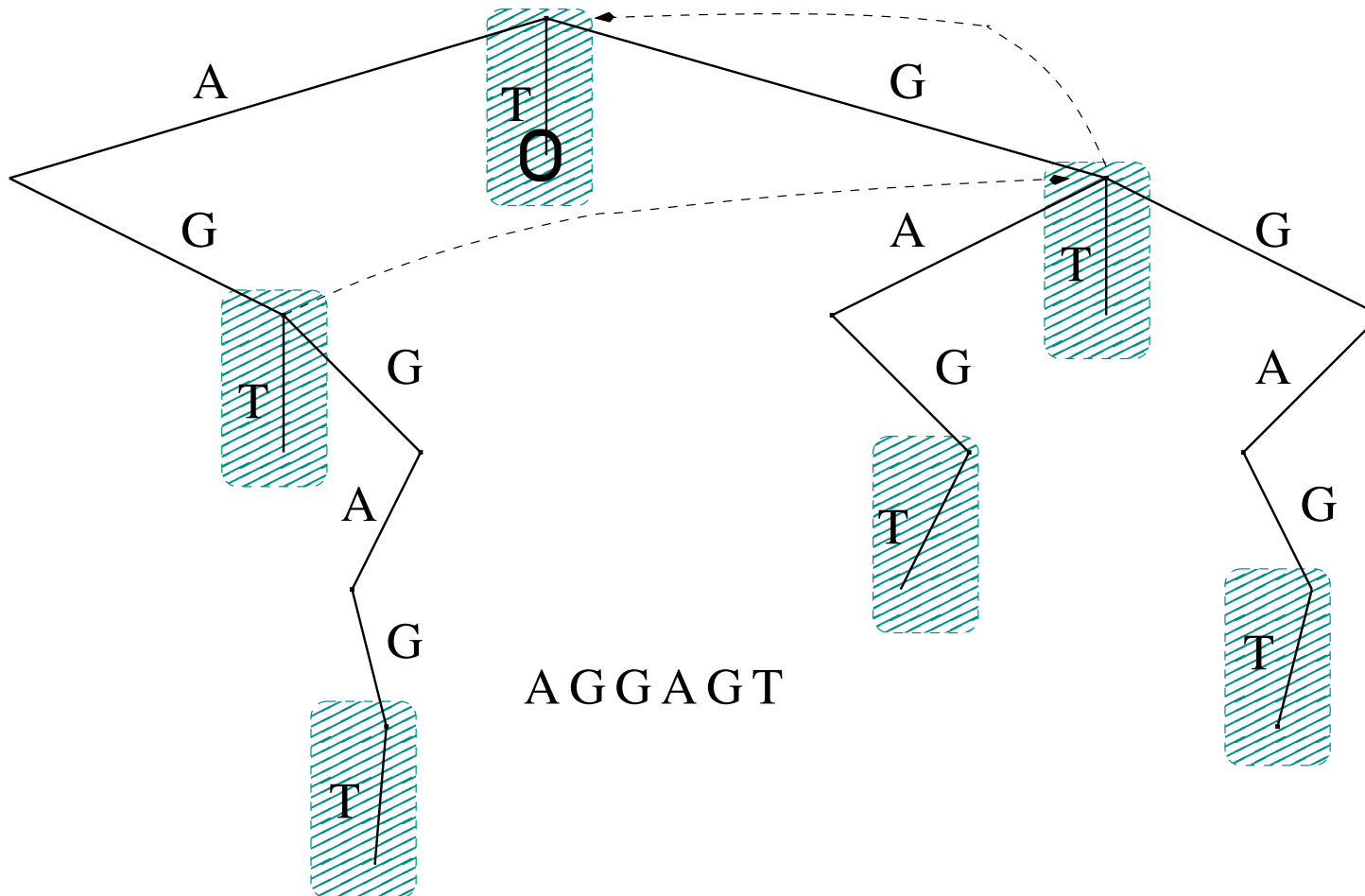
# Suffix tree construction

Adding  $T$  to  $AGGAG$ , fast insertion of  $T$  from the root



# Suffix tree construction

remark : leaves are consecutively created at each level

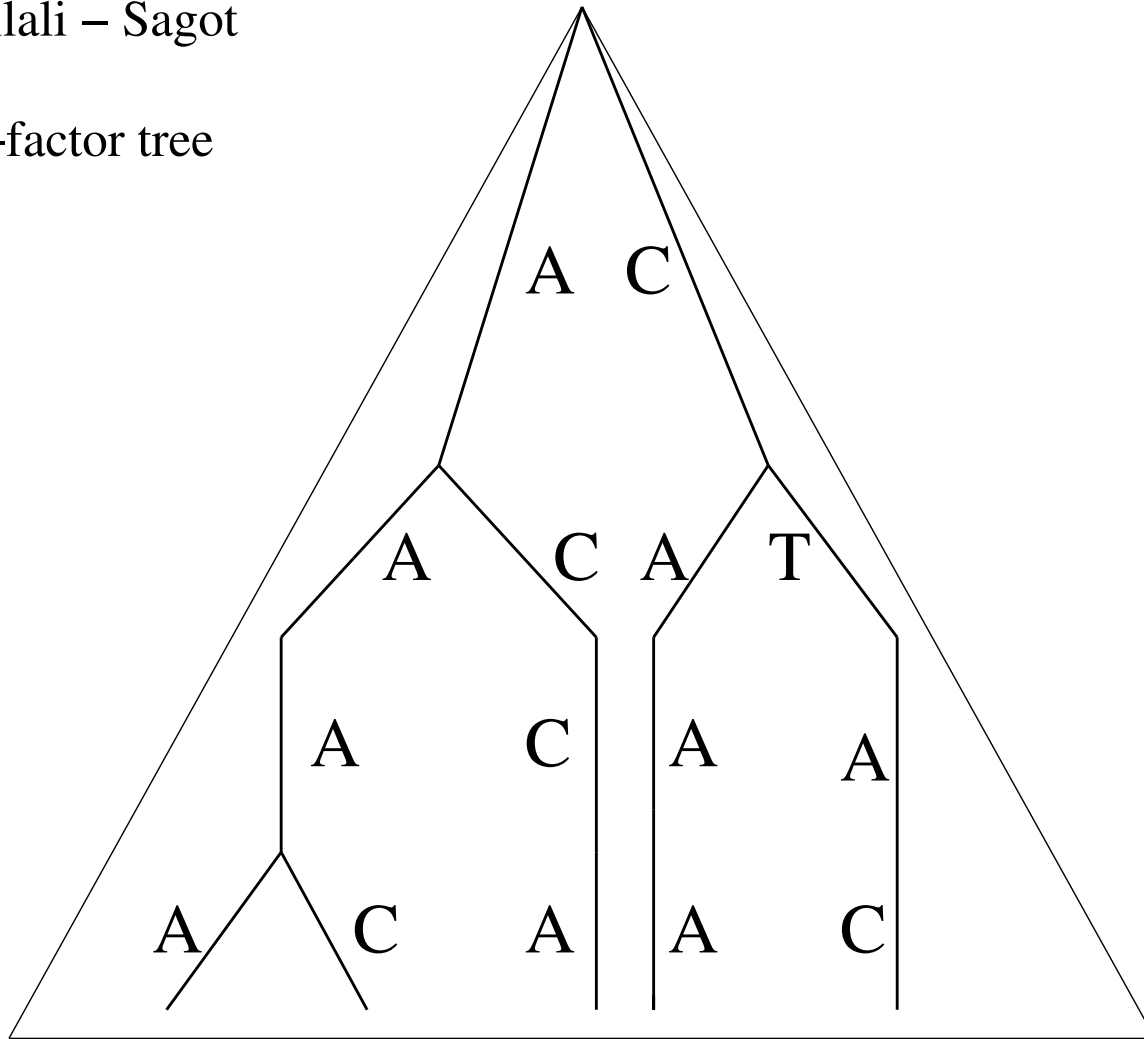




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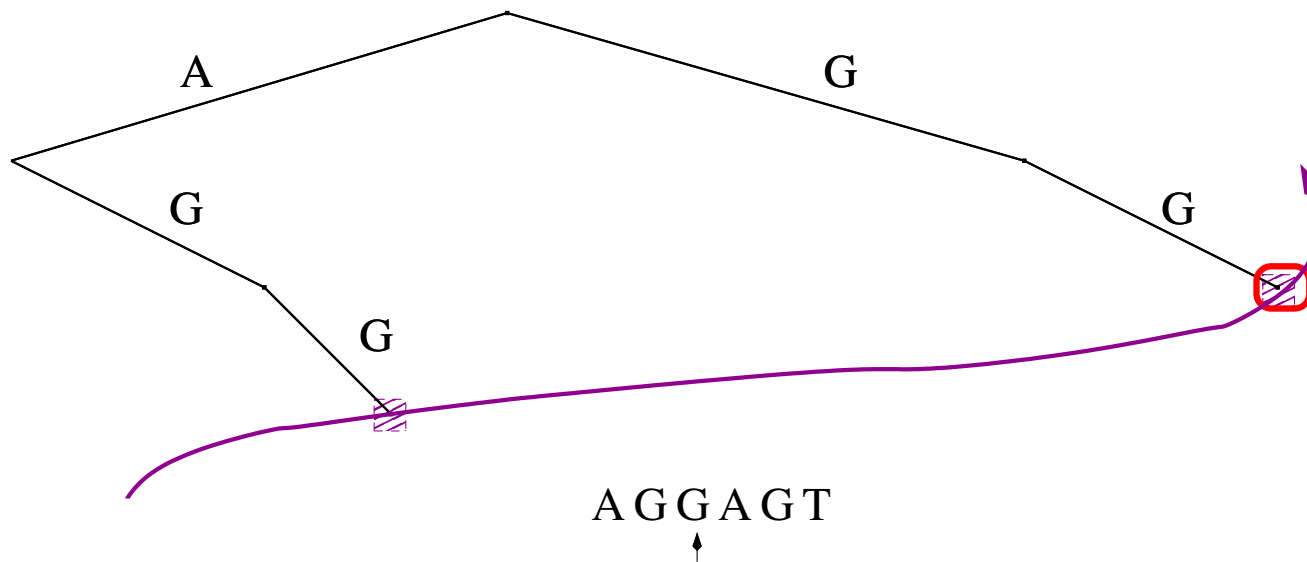
Allali – Sagot

k-factor tree



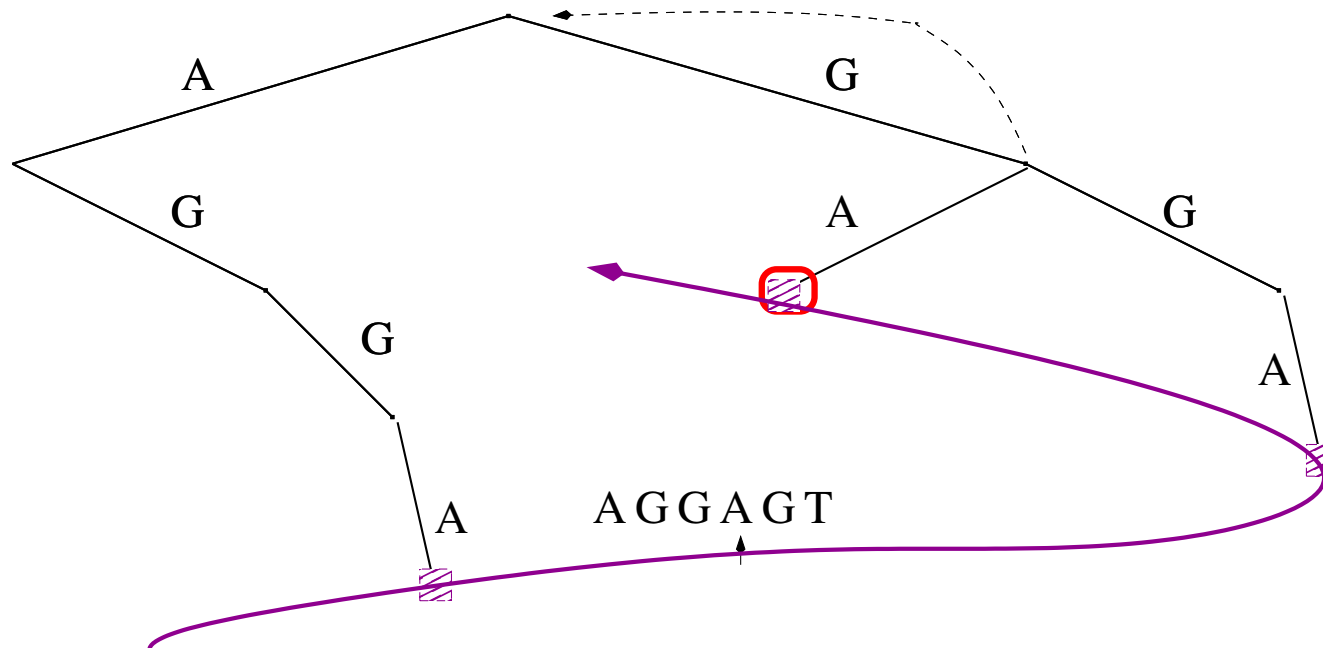
# $k$ -factor tree construction ( $k = 4$ )

$k - 1$  first phases : usual construction of a suffix tree, putting the leaves in a queue



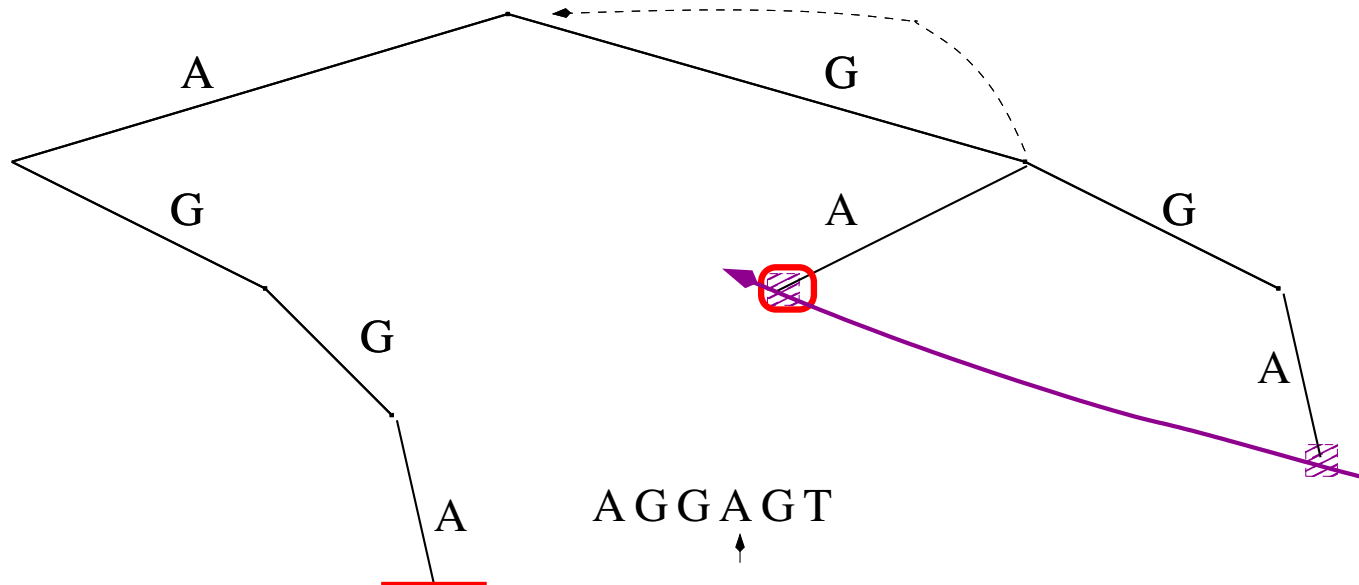
## $k$ -factor tree construction ( $k = 4$ )

After  $k^{\text{th}}$  phase, usual construction of a suffix tree, still putting the leaves in a queue, but...



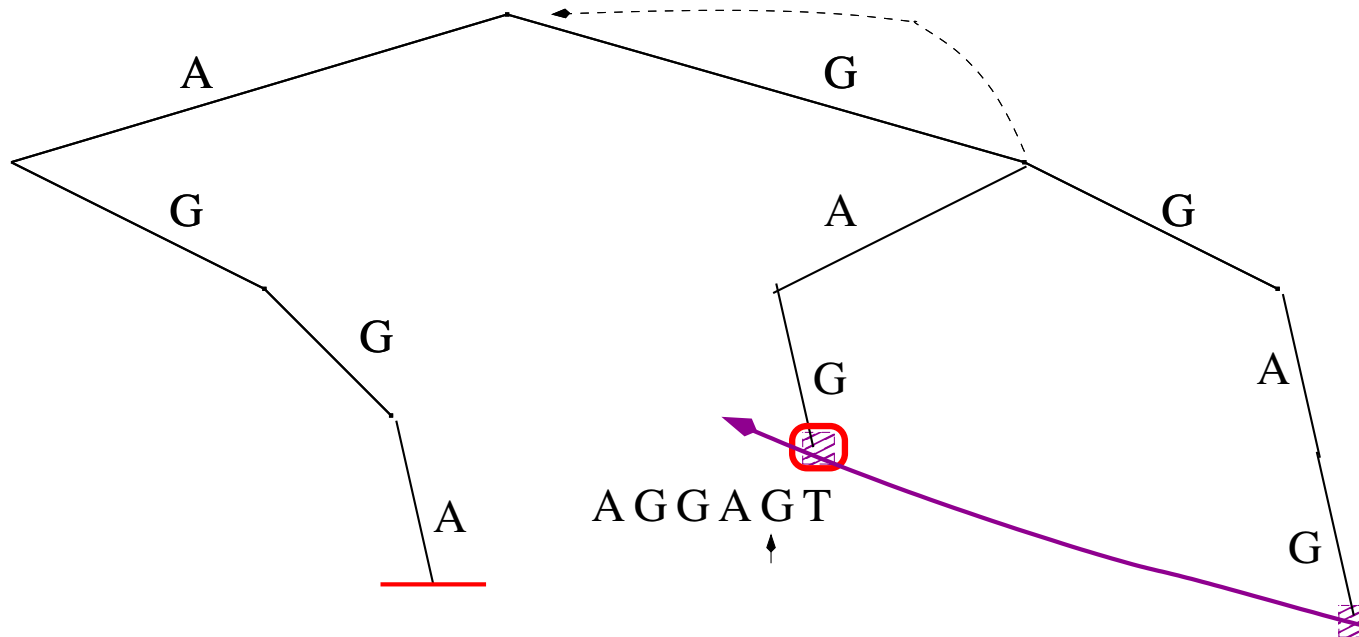
## $k$ -factor tree construction ( $k = 4$ )

...but removing the end of the queue at the end of the phase, stopping the automatic extension of the leaving leaf



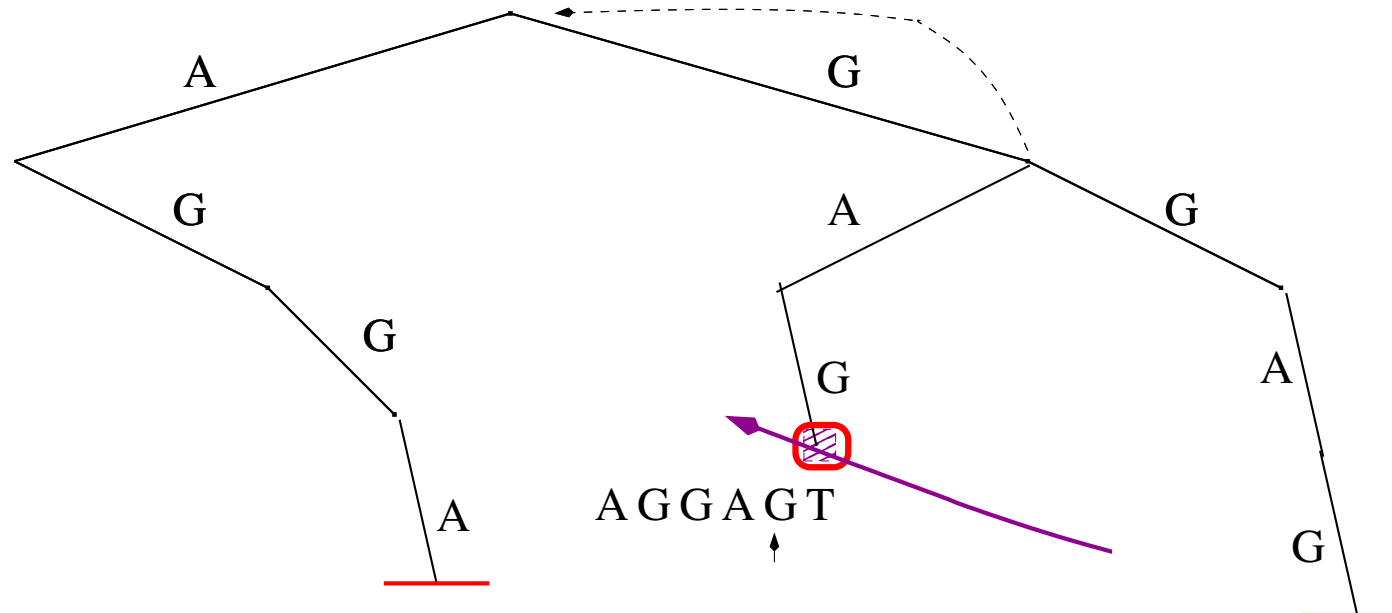
# $k$ -factor tree construction ( $k = 4$ )

Next step,



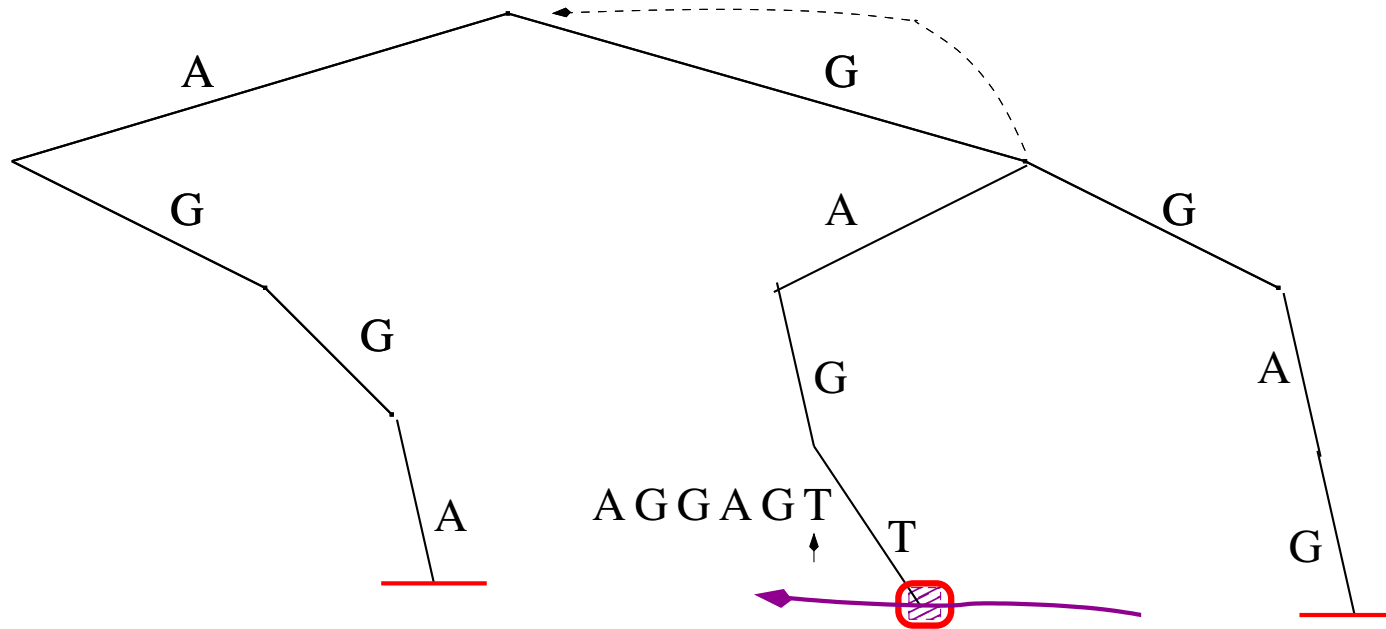
# $k$ -factor tree construction ( $k = 4$ )

Next step, remove head



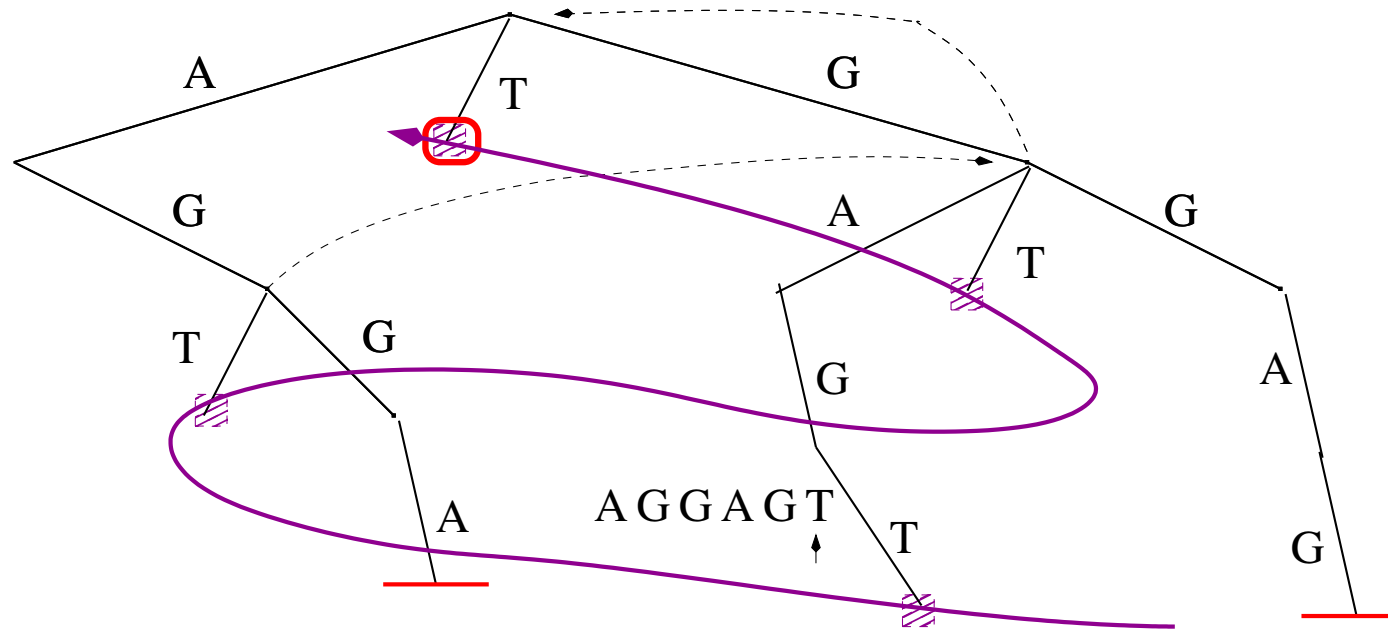
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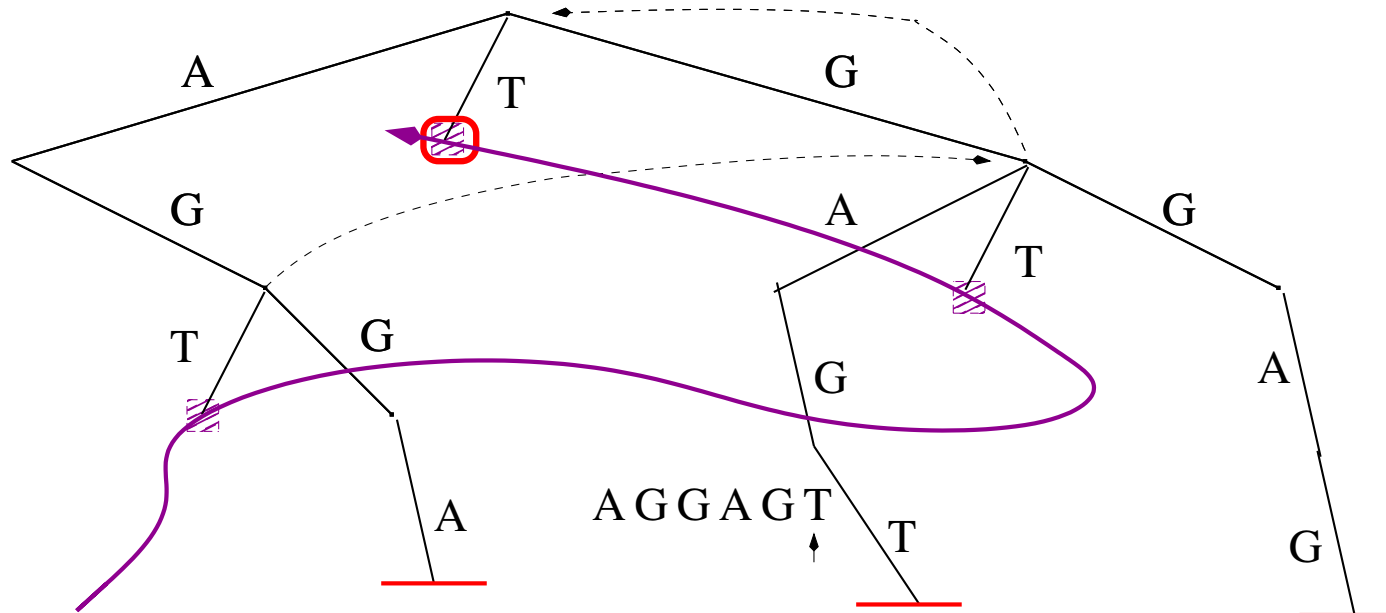
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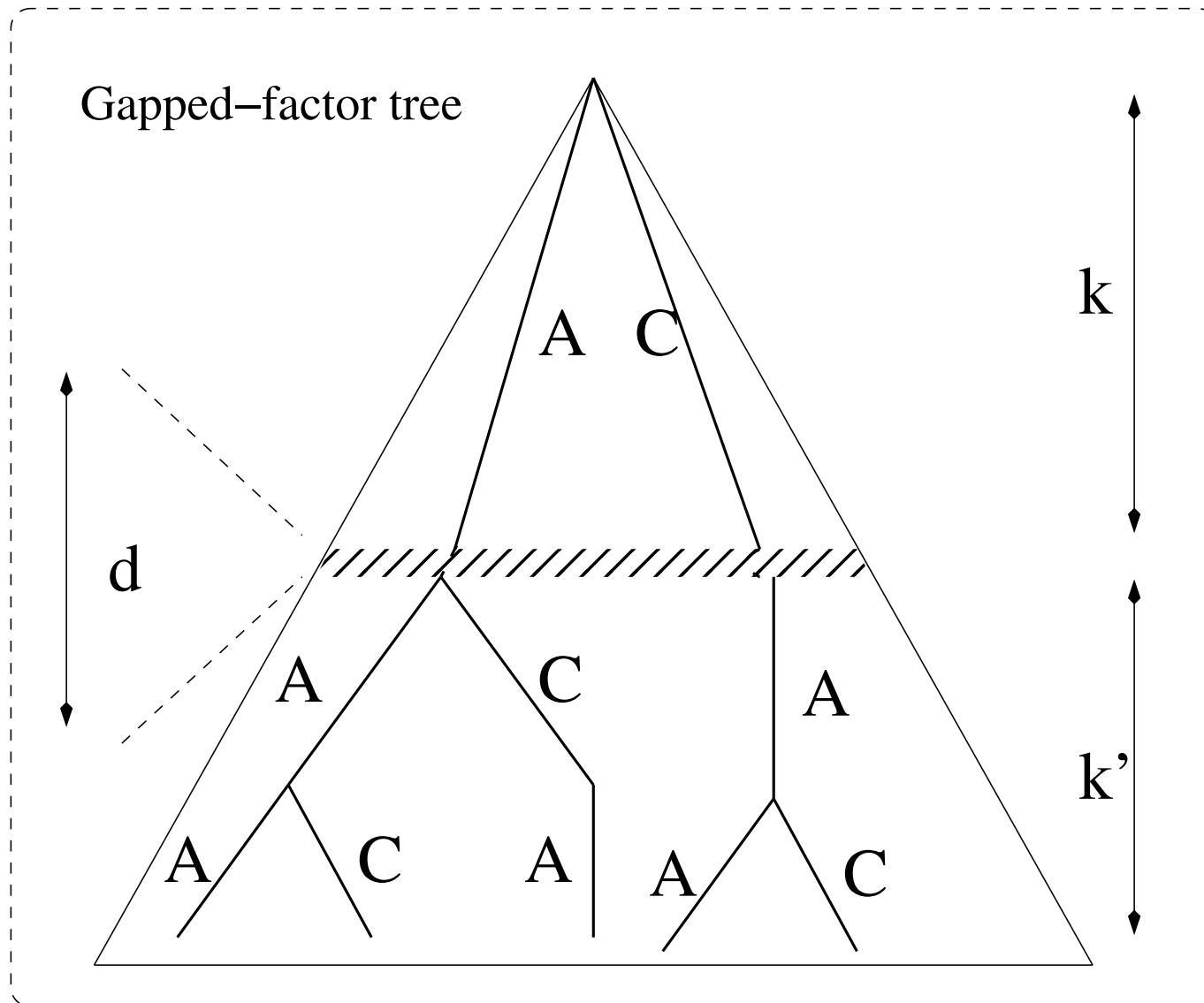
Construction

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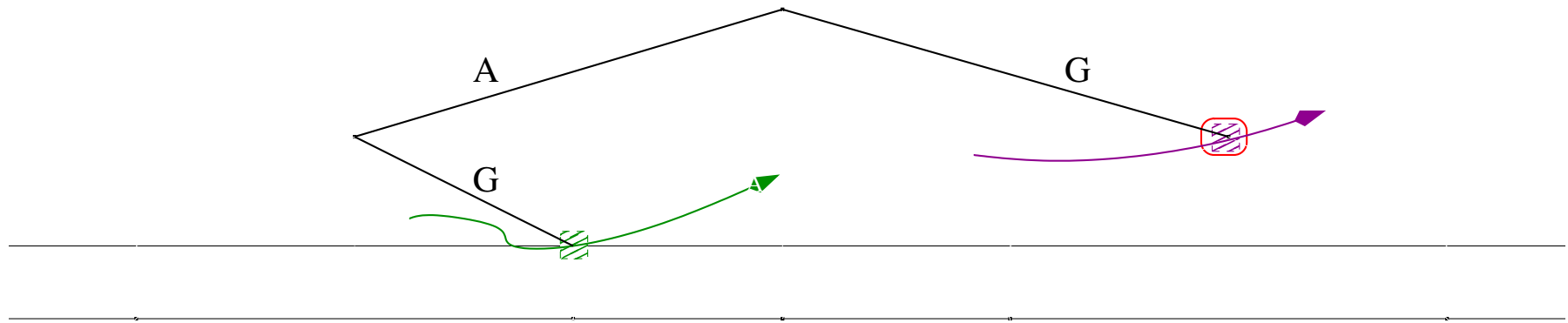


## Overview



# Gapped-factor tree construction ( $k = 2, d = 1, k' = 3$ )




Three queues : upper extension leaves , hidden extension leaves ,

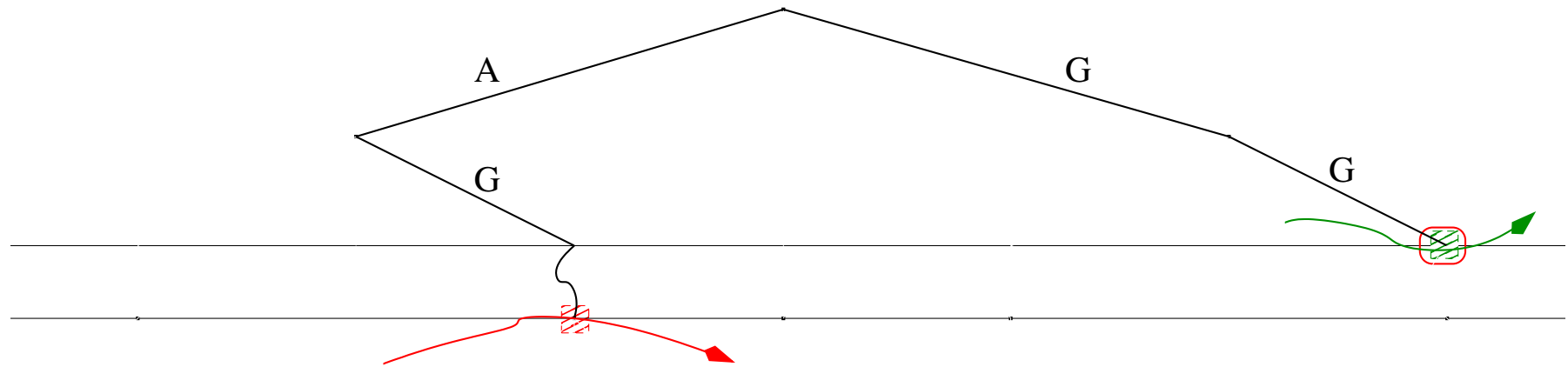


AGGAGAACAA



# Gapped-factor tree construction ( $k = 2, d = 1, k' = 3$ )

Three queues : upper extension leaves , hidden extension leaves , lower extension leaves 

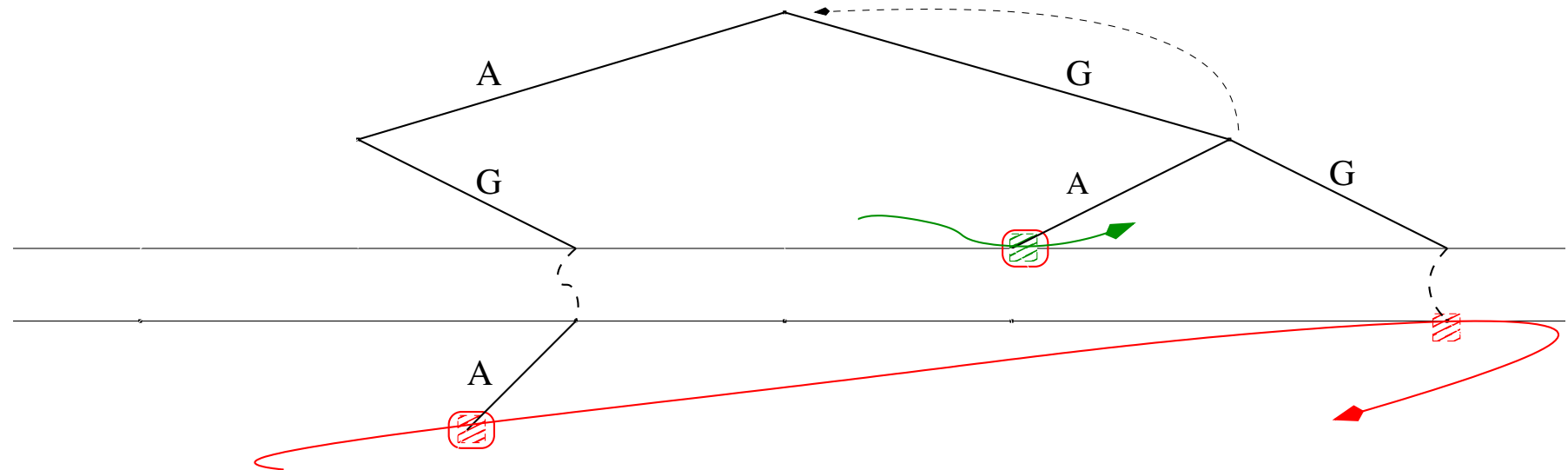


AGGAGAACAA



# Gapped-factor tree construction ( $k = 2, d = 1, k' = 3$ )

Lower part of the tree : same principles as for the upper part

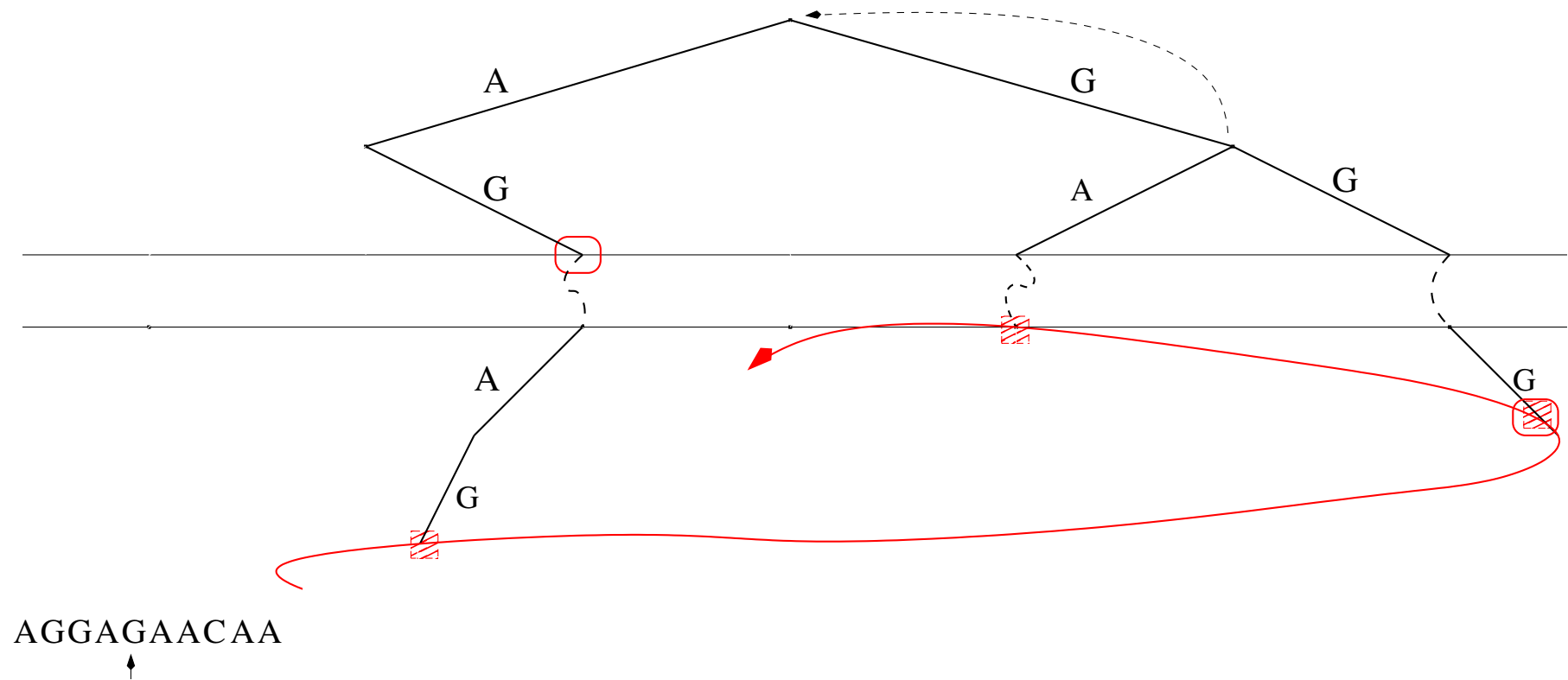


AGGAGAACAA



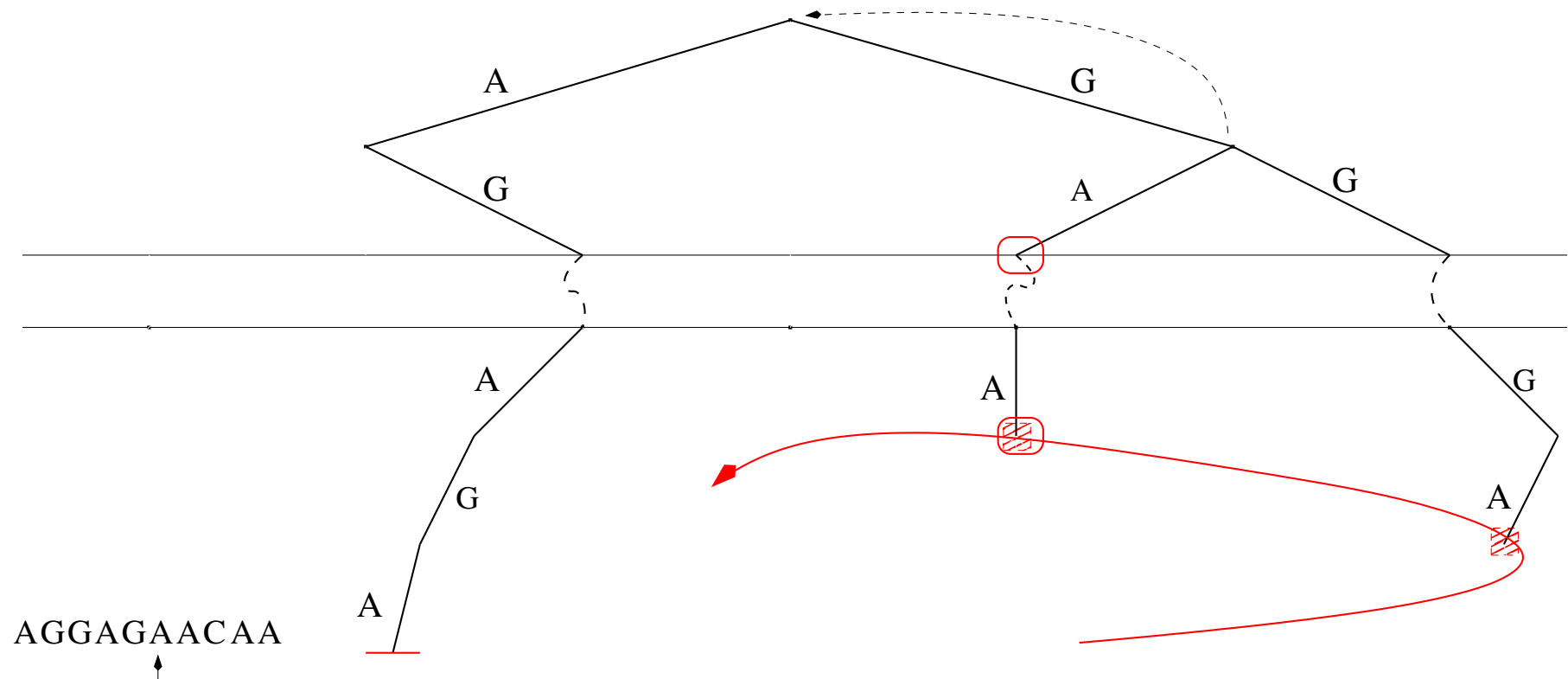
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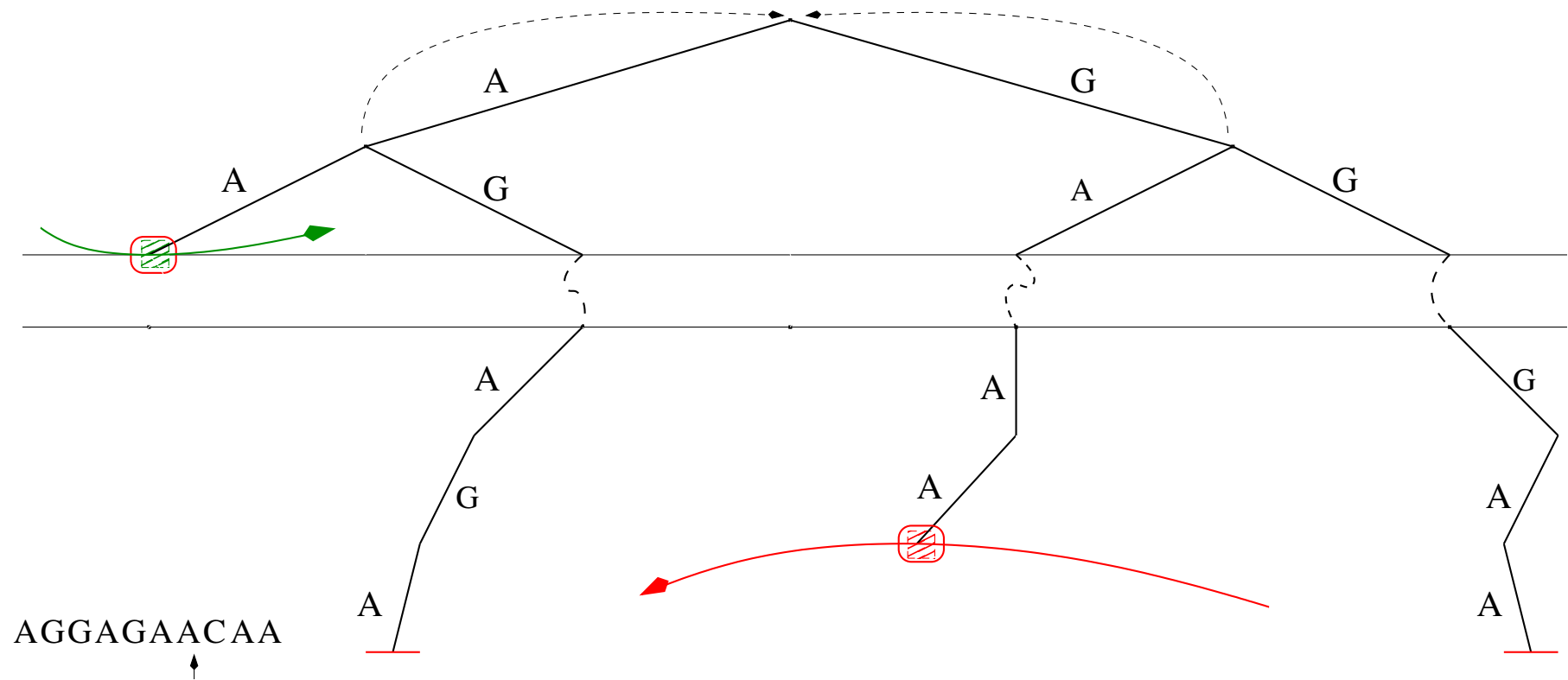
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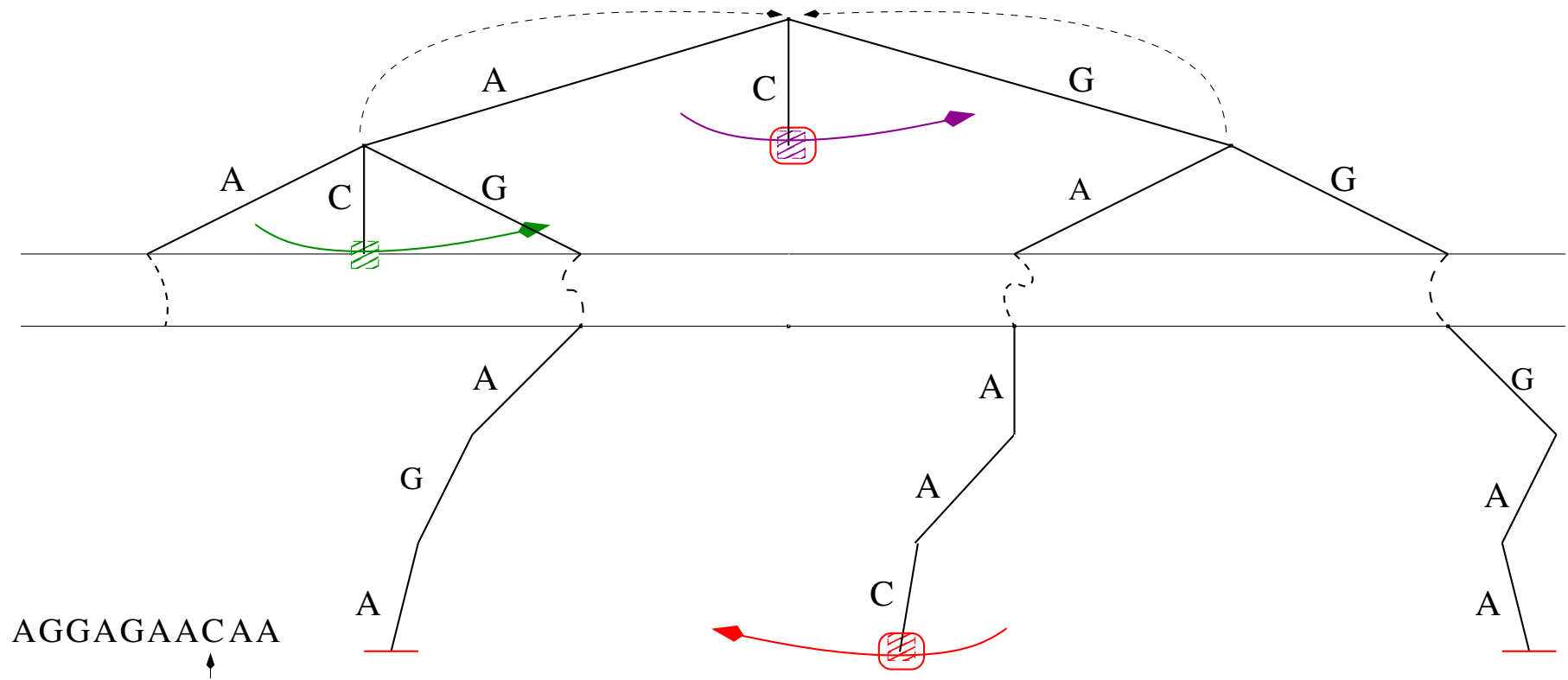
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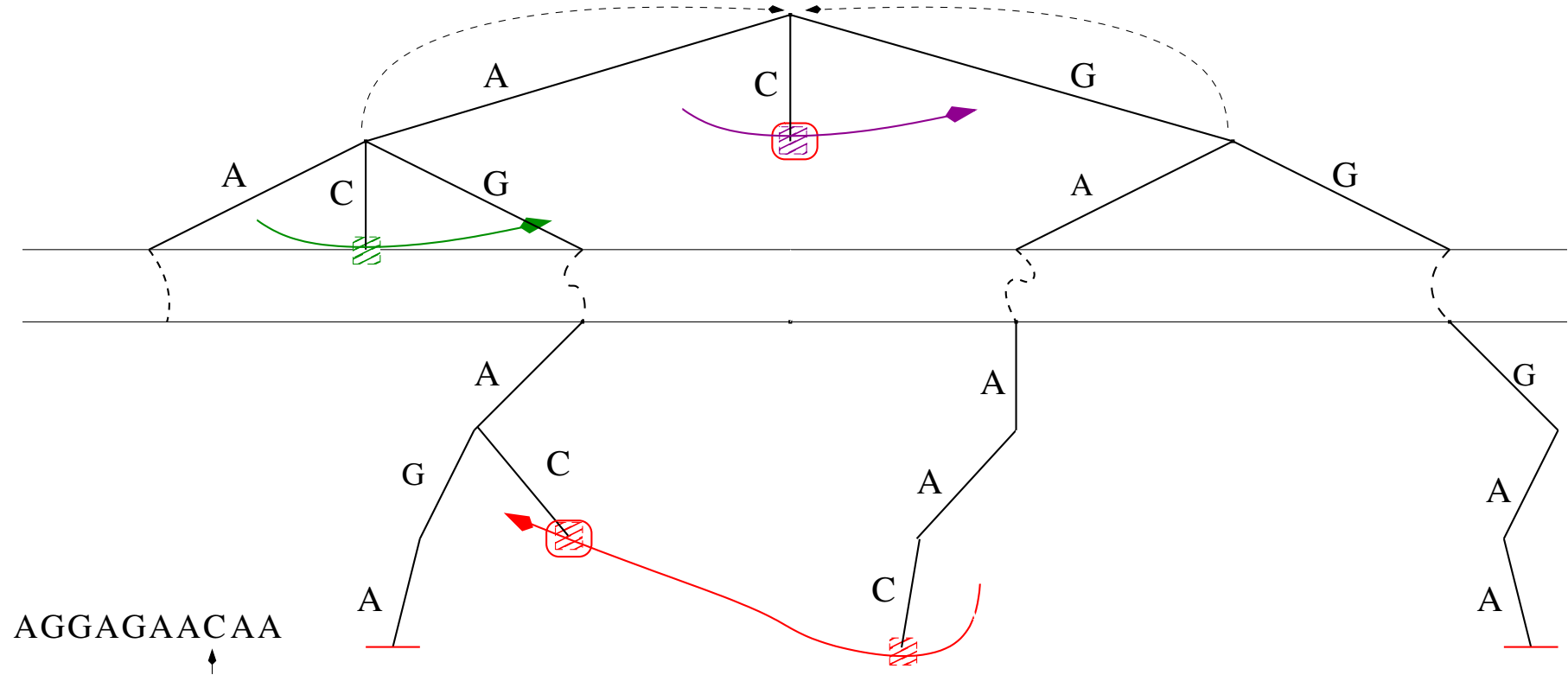


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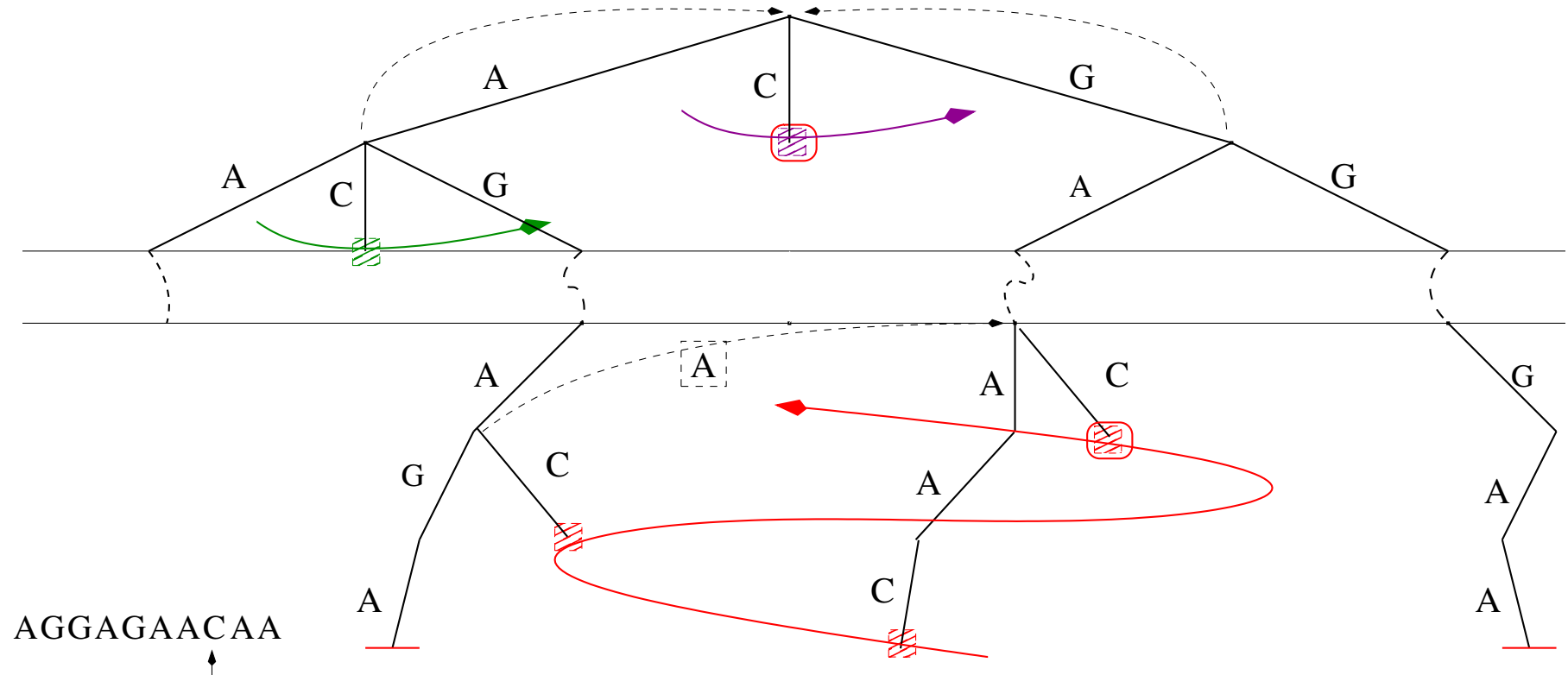
Multiple suffix link



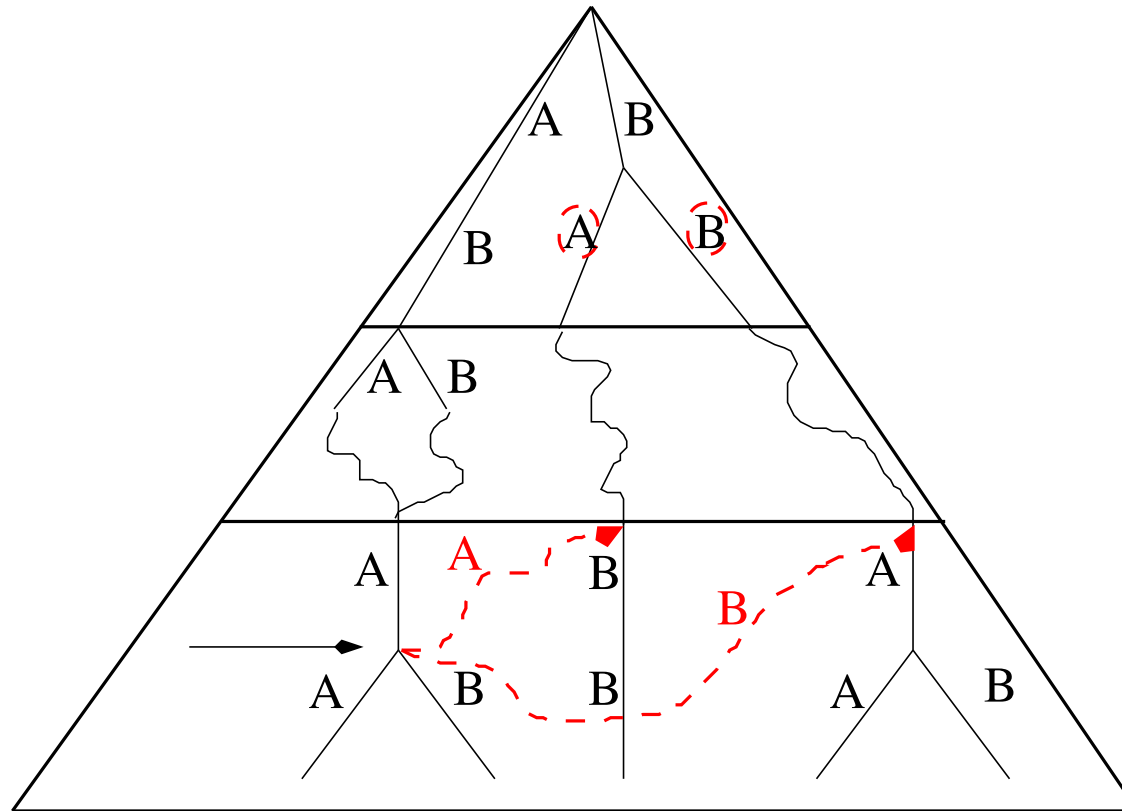
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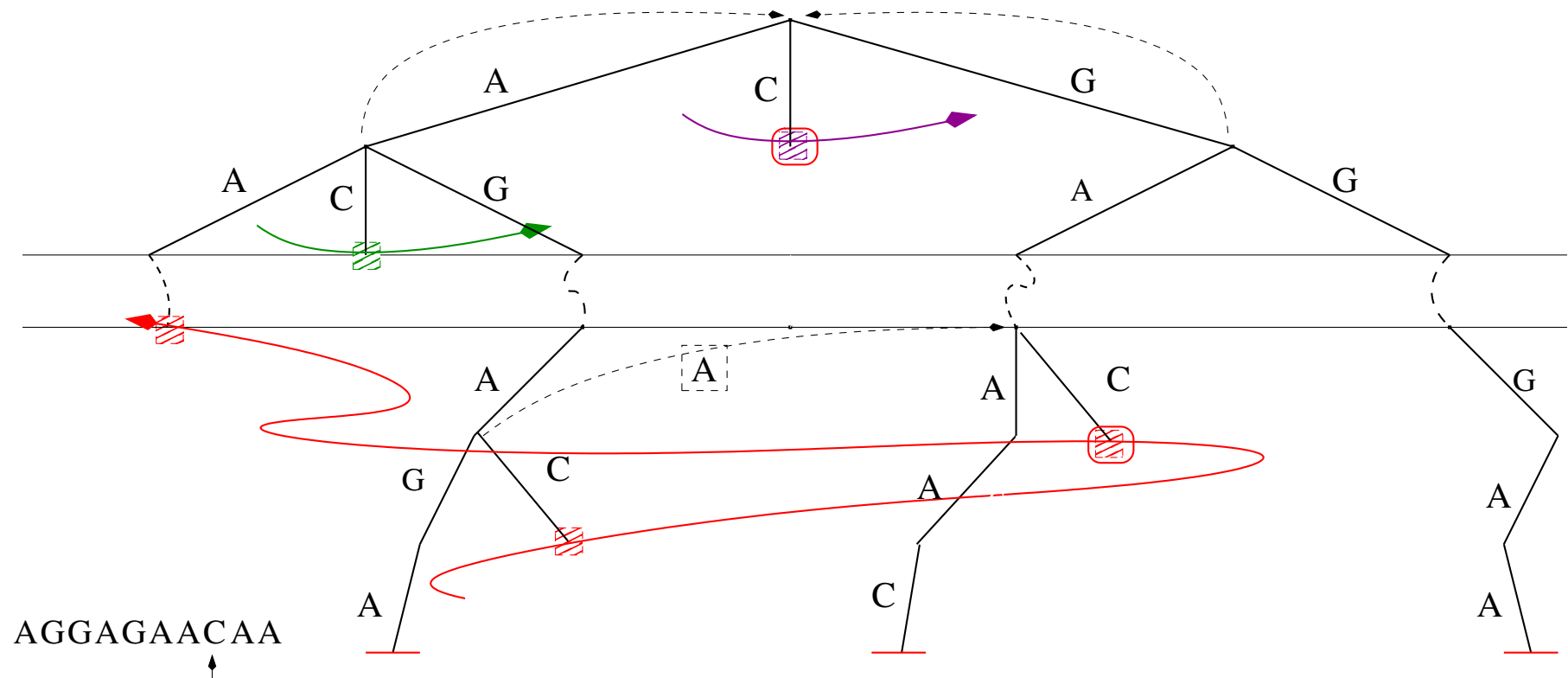
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# Complexity analysis

## Time and memory

- During the construction :  $O(n \times |\Sigma|)$
- Using the index :  $O(n)$



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- Interesting properties on suffix trees
- Use and development of Ukkonen's method
- Indexing structure useful for various stringology problems



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- Interesting properties on suffix trees
- Use and development of Ukkonen's method
- Indexing structure useful for various stringology problems
- Gapped suffix array  $O(n)$

