A Concurrent Specification of Brzozowksi's DFA Construction Algorithm

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The Prague Stringology Conference '06

Sequential algorithm

$\delta, S, F := \varnothing, \{E\}, \varnothing;$
$D, T := \varnothing, S;$
do $(T \neq \varnothing) \rightarrow$
  let q be some state such that $q \in T$;
  $D, T := D \cup \{q\}, T \setminus \{q\};$
  $\{$ *build out-transitions from q on all alphabet symbols* $\}$
  for $(i : \Sigma) \rightarrow$
    $\{$ *find derivative of q with respect to i* $\}$
    $d := i^{-1}q;$
    if $d \notin (D \cup T) \rightarrow T := T \cup \{d\}$
    $[\!]$ $d \in (D \cup T) \rightarrow$ skip
    fi;
    $\{$ *make a transition from q to d on i* $\}$
    $\delta(q, i) := d$
  rof;
  if $\varepsilon \in \mathcal{L}(q) \rightarrow F := F \cup \{q\}$
  $[\!]$ $\varepsilon \notin \mathcal{L}(q) \rightarrow$ skip
  fi
od;
return $(D, \Sigma, \delta, S, F)$

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Selected CSP notation

$a \rightarrow P$	event a then process Q
$a \rightarrow P \vert b \rightarrow Q$	a then P choice b then Q
$x : A \rightarrow P(x)$	choice of x from set A then $P(x)$
$P \parallel Q$	P in parallel with Q
	Synchronize on common events in alphabets
$b!e$	on channel b output event e
$b?x$	from channel b input to variable x
$P \not< C \not> Q$	if C then process P else process Q
$P; Q$	process P followed by process Q
$P \Box Q$	process P choice process Q

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The BRZ process

$$BRZ(D, T) = OUTER(D, T) \parallel DERIVE \parallel UPDATE$$

- OUTER corresponds with outer loop.
- DERIVE caters for the computation of derivatives.
- UPDATE caters for the determination of which regular expressions should be used to update T and also for updating δ.

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The OUTER process

- Some $q \in T$ is selected to build its outgoing transitions.
- Updating of set D, T, and F.

$$OUTER(D, T) =$$

$$(q : T \rightarrow (EXTRACT(q) \parallel FINAL(q)); OUTER(D \cup \{q\}, T \setminus \{q\}))$$

$$\Box$$

$$(insert?q \rightarrow OUTER(D, T \cup \{q\}))$$

The EXTRACT and FINAL processes

- Broadcasts the selected q to the DERIVE processes.

$$EXTRACT(q) = \|_{i:\Sigma}\ EXTRACT_i(q) \quad \text{where}$$

$$EXTRACT_i(q) = (dIn_i!q \rightarrow SKIP)$$

- Updates F, if needed.

$$FINAL(q) = F := F \cup \{q\} \not< \varepsilon \in \mathcal{L}(q) \not> SKIP$$

The DERIVE process

- Finds the derivatives of a regular expression in parallel.

$$DERIVE = \|_{i:\Sigma} DERIVE_i \quad \text{where}$$

$$DERIVE_i = dIn_i?q \rightarrow dOut_i!(q, \frac{d}{di}q) \rightarrow DERIVE_i$$

The UPDATE process

- Receives derivatives and updates δ.
- Feeds back the derivative to OUTER.

$$UPDATE = \|_{i:\Sigma}\ UPDATE_i$$

$$UPDATE_i = (dOut_i?(q,d) \rightarrow (UPT_i(d) \parallel UPD_i(q,d))\ ); UPDATE_i$$

$$UPT_i(d) = insert!d \rightarrow SKIP \lhd d \notin (D \cup T) \rhd SKIP$$

$$UPD_i(q,d) = \delta(q,i) := d$$

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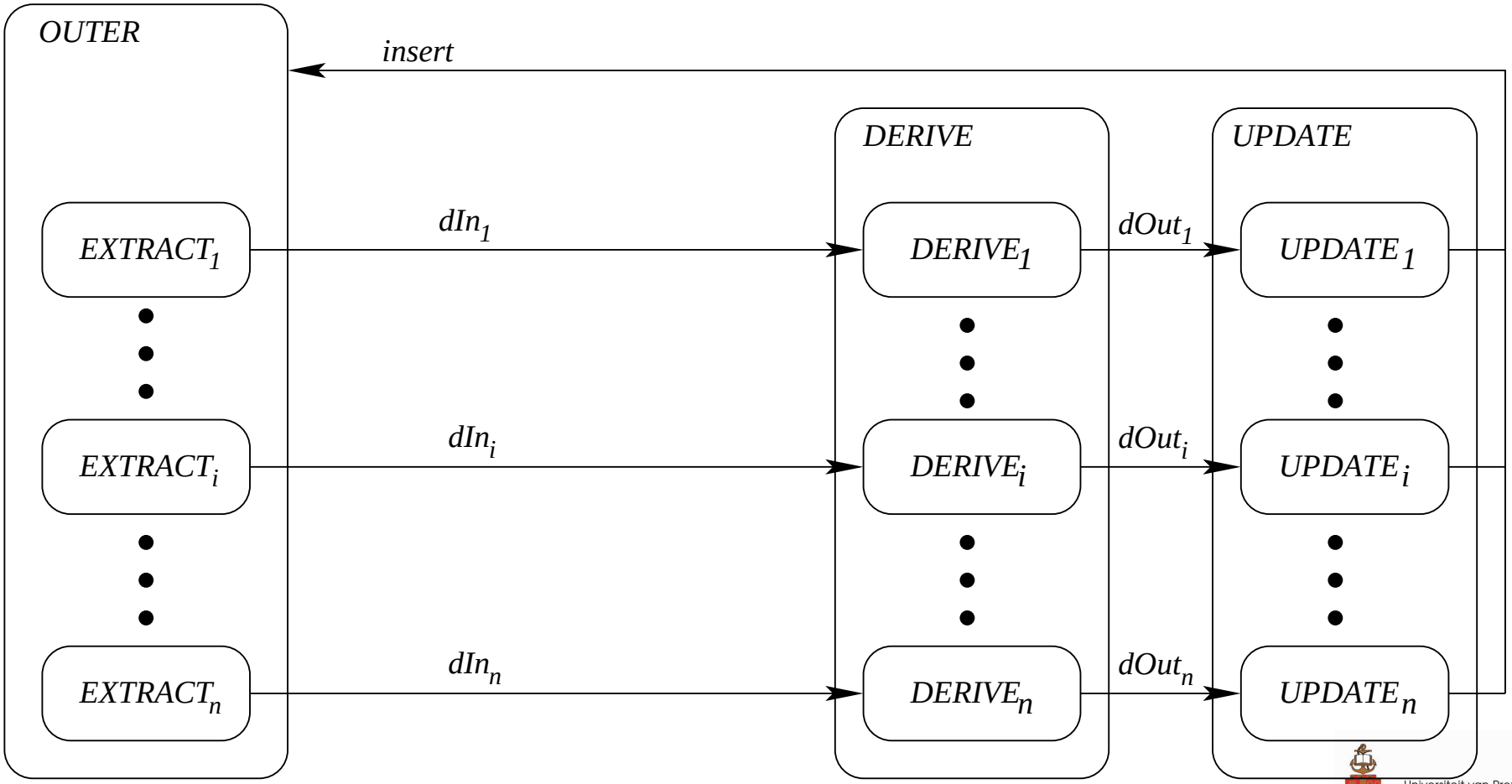
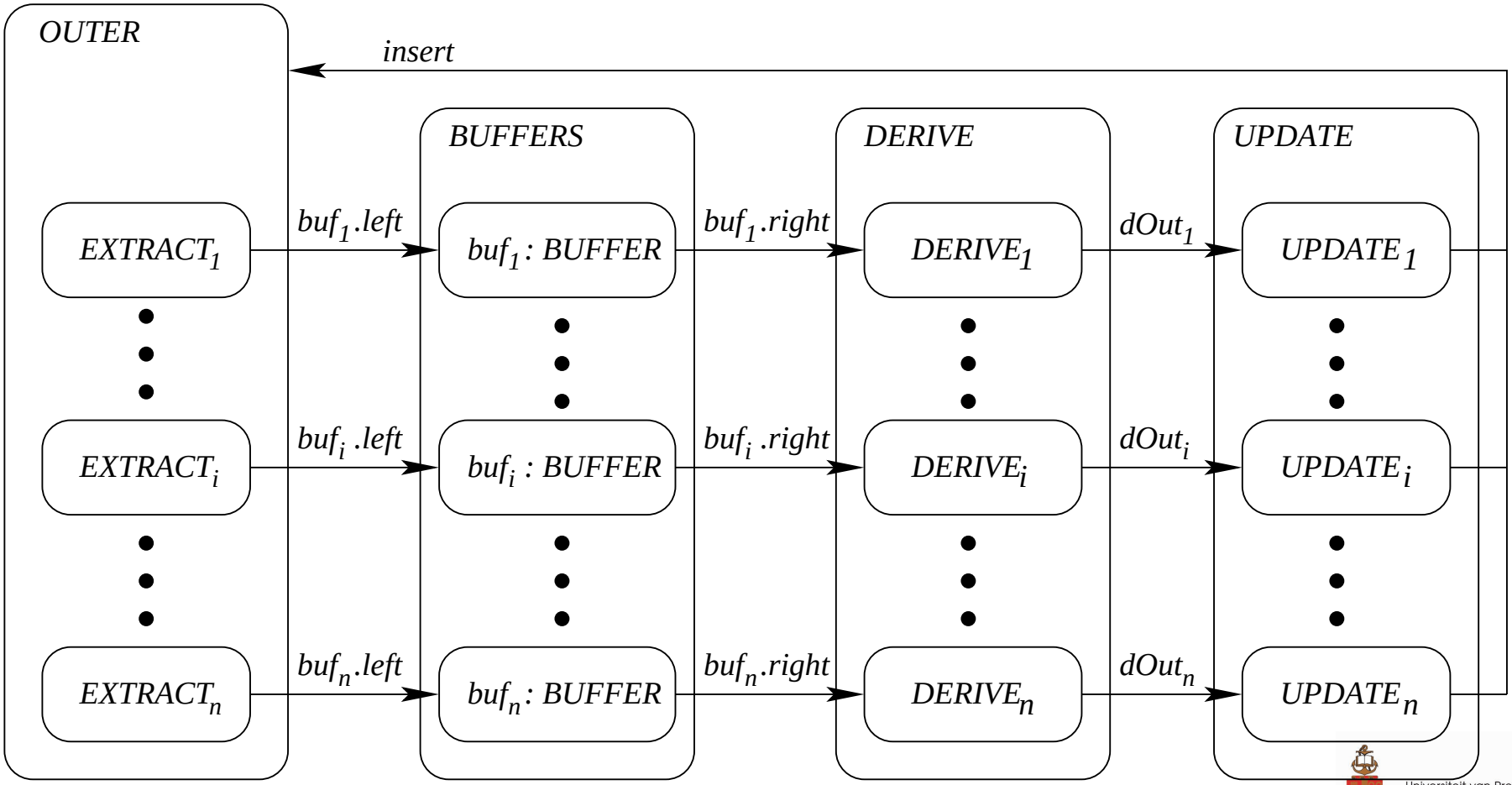
Graphical representation



Buffered version



Conclusion

- Presented an abstract concurrent specification.
- Attempted to allow for as much concurrency as possible.

- Next steps
  - Implement concurrent Brzozowski.
  - Other FA algorithms such as minimisation.